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UNCOVERING THE COMMON RISK FREE RATE IN THE EUROPEAN MONETARY UNION

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Abstract

We introduce Longitudinal Factor Analysis (LFA) to extract the Common Risk Free (CRF) rate from a sample of sovereign bonds of countries in a monetary union. Since LFA exploits the typically very large longitudinal dimension of bond data, it performs better than traditional factor analysis methods that rely on the much smaller cross-sectional dimension. European sovereign bond yields for the period 2006-2010 are decomposed into a CRF rate, a default risk premium, and a liquidity risk premium, shedding new light on issues such as benchmark status, flight-to-quality and flight-to-liquidity hypotheses. Our empirical findings suggest that investors chase both credit quality and liquidity, and that liquidity is more valued when aggregate risk is high.

JEL Classification codes: C19, E43, G12

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1 Introduction

The risk free interest rate is a corner-stone in the pricing of financial assets, risk measurement, and inter-temporal allocation models. For a monetary union, the risk free rate is not as easily observable as for countries with an individual currency. To address this point, this paper introduces a new technique to extract the *common risk free rate* from a sample of sovereign bonds.

In general the risk free interest rate, or in short the risk free rate, is defined as the return that can be obtained by investing in (short-term) financial instruments with no default risk. For example, default on US treasury bills is theoretically impossible because the US government can repeal the Federal Reserve's independence and have as much money printed as needed to honour its financial obligations. In contrast, in a monetary union with centralised monetary policy and decentralised tax collection, default-free instruments do not exist.

In this paper the Common Risk Free (CRF) rate represents the return on a hypothetical common bond without default and liquidity risk. The CRF rate equals the minimum possible aggregate nominal funding costs of the union's member states, and reflects the fundamentals of the union's economy. Since we analyse long-term instruments, the CRF bond is however not free of inflation and market risk (*i.e.* the variability of short-term interest rates). The CRF rate includes the common part of the inflation risk premium across the members of the monetary union. Adjusting the bond yields for cross country differences in this premium is unnecessary because international investors are not affected by inflation outside their country of residence.

Mayordomo et al. (2009) derive the CRF rate by using macro-economic variables such as the debt to GDP ratio to first estimate the country specific risk premiums. The CRF rate is then computed as the average of the countries' bond yields adjusted for the corresponding risk premiums. We do the opposite by first estimating the CRF rate and then derive the risk premiums. Risk premiums in turn can be decomposed in a default risk premium and a liquidity risk premium.[1] None of the three bond yield components are directly observable, as even the sovereign Credit Default Swap (CDS) rate cannot be taken as a direct measure of the corresponding bond credit

[1] Liquidity risk arises from uncertainty about deadweight losses when a security is sold before its expiration date.

risk premium, as suggested by Mayordomo et al. (2009), since it also contains its own liquidity risk component.

We attribute the first common factor in the CDS-adjusted sovereign bond yields to the CRF rate. Note that the deviations of the CDS-adjusted bond yields from the CRF rate consist of differences between, on the one hand, implied credit and liquidity risk premiums on the bond, and, on the other hand, implied credit and liquidity risk premiums on the CDS. Common factors in these differences across countries are assumed to be less important than the CRF rate for the variation in the bond yields. This holds when both the difference in the implied price of credit risk and the difference in liquidity risk premiums between the bond and the CDS are small relative to the CRF rate. As shown below, usually such a situation occurs.

A new method for factor analysis is introduced to extract this unobserved common component. Like with classical factor analysis (see Jöreskog, 1969) our method finds the common component up to an additive and multiplicative scaling factor. However, these scaling factors can be derived under weak assumptions. One assumption is the existence of a benchmark security. Benchmark securities are used for price discovery of market-wide phenomena (see Hasbrouck, 1996). In the absence of benchmark bond specific news (such as a deterioration of credit quality) investors attribute price changes of the benchmark bond fully to the risk free rate. The sensitivity of the benchmark bond to the CRF rate thus equals unity. This does not necessarily imply that all risks inherent in the benchmark security are systematic as suggested by Yuan (2005) and Dunne et al. (2007). In our model, only variability in the risk free rate leads to systematic risk. Although the benchmark bond may carry a time-varying risk premium, by assuming that the benchmark bond tracks the risk free rate, the multiplicative factor for the re-scaling of the common component follows from the factor loading associated with the benchmark bond. The additive factor follows by leaving no unexplained fixed components in the deviations of the CDS-adjusted bond yields from the CRF rate, once, these deviations, in turn, are corrected for differences in bond and CDS liquidity risk. Here we assume that the liquidity risk premium, *i.e.* the price of possible future transaction costs, is positively and linearly related to current transaction costs. In principle, risk premiums could be negative due to preferential tax treatment even when economic agents are risk averse. Landon (2009) however finds that since 1994 taxes have not been capitalised in Canadian government bond yields, suggesting that the marginal investor is best represented by a non-taxed entity. Accordingly, in this paper we ignore the possible effects of taxes. Amihud and Mendelson (1991) confirm the pricing of a liquidity effect in US treasury notes and bills.

In order to improve the efficiency of factor analysis we derive cross-variable restrictions, and maximize the likelihood function under these restrictions. Classical factor analysis ignores these restrictions. Since our method, hereafter called Longitudinal Factor Analysis (LFA), exploits the typically very large longitudinal dimension of bond data, it performs better than traditional factor analysis methods that rely on the much smaller cross-sectional dimension. The results of a Monte Carlo experiment show that LFA is more efficient in the estimation of idiosyncratic risk and the common component than Principal Components (PC) based on Theil's (1971) method and classical factor analysis based on the EM algorithm of Rubin and Thayer (1982). Both PC and EM substantially overestimate the idiosyncratic risk on relatively low risk bonds, while conversely the risk on high risk bonds is systematically underestimated. The deviations are up to 73% and 27% of the true risk for the PC and EM methods respectively. In contrast, LFA only slightly overestimates the idiosyncratic risk of all bonds with less than 0.4% of the true risk. So far, the use of our estimation technique is limited to models with one common factor only.

Bond and CDS data for the European Monetary Union (EMU) are used to derive the 5- and 10-year CRF rate in the euro area during the years 2006 till 2010. In addition to eleven sovereign issuers we also include bonds of the European Investment Bank (EIB). Since the EIB is owned by the member states of the European Union, all EMU countries are liable for these bonds. EIB bonds are however different from a common EMU bond since the liability of EIB owners is limited to their amount of subscribed capital, which is below the debt outstanding, and some countries outside the EMU are also liable for EIB bonds.

Finance professionals generally consider the German Bund as the benchmark for the euro-denominated sovereign bond market. Based on arguments related to price discovery, academics have suggested other possible definitions of benchmark status such as the asset with the lowest idiosyncratic risk (see Dunne, Moore and Portes (2007), henceforth referred to as DMP, for evidence on EMU sovereign benchmarks).

Our base-case uses the Bund as the *a priori* benchmark. Robustness checks are made by using alternatively one of the other countries' bonds as *a priori* benchmark.

Analyzing French, German and Italian bonds, DMP designate, with the exception of very long-term bonds, French bonds as sovereign benchmarks for the period April 2003 – March 2005 because bi-lateral inter-bond regressions with French bonds have lowest residual variances. Our empirical results are in accordance with this finding for the pre-crisis period up to June 2007 included, but not thereafter. Moreover, before the crisis, there are other bonds, such as Dutch 5-year bonds that are even less risky than French 5-year bonds. Since the sub-prime mortgage crisis, however, Bunds have the lowest risk. The lowest variance in the total risk premium is observed for the *a priori* benchmark only if the Bund is chosen as the benchmark. For all other choices of benchmark, the Bund remains the bond with the lowest idiosyncratic risk. Hence, the data confirm its benchmark status.

Our method for determining which bond has the lowest idiosyncratic risk has two important advantages over the one proposed by DMP. First, it can be applied for any number of assets whereas the DMP method cannot handle more than three. Secondly, in Section 3 we show that the inter-bond regressions cannot be run independently since the errors are not independent across equations. Our method solves this problem by maximising the likelihood function with the errors of the structural equations only.

Our main other empirical findings are as follows: 1) from the moment the financial crisis hit the sovereign bond markets, the German Bund does not correspond closely to the common risk free rate any longer. Investors have begun to demand a significant risk premium even on benchmark securities. 2) The increase in credit risk premiums is by far the dominant factor in the divergence of the euro area sovereign bond yields. The increase in the liquidity risk premiums is relatively small and only plays a minor role for all bonds. 3) Since the crisis, bonds with higher credit risk also tend to have higher liquidity risk. Furthermore, liquidity risk and credit risk are positively correlated over time, meaning that liquidity is more valued during episodes of higher aggregate risk. 4) Sovereign credit risk priced in euro area bonds tend to be higher than that priced into credit default swaps, suggesting that the derivative markets are not driving up bond yields. In the main text we compare these results to the findings of recent other studies.

The remainder of this paper is organised in four parts: Section 2 explains how the common risk free rate in a monetary union can be uncovered by exploiting the commonalities in bond yields of different sovereigns. Section 3 discusses the new statistical method to find this common factor. Section 4 decomposes the sovereign bond yields in the European Monetary Union in the common risk free rate, the default risk premium and the liquidity risk premium, and analyses how these components have behaved during the recent crisis. Section 5 concludes.

2 A parsimonious model for the common risk free rate in a monetary union

Let y_{it} be the sovereign bond yield of member country i of the monetary union at time t for a particular maturity. y_{it} can be decomposed in a risk free component (R_t) common to all n countries in the union, and a country specific risk premium (π_{it}):

$$y_{it} \equiv R_t + \pi_{it},\ i \in N, t \in \Gamma, \tag{1}$$

where $N \equiv \{1,...,n\}$ and $\Gamma \equiv \{1,...,T\}$. The two components of the risk premium (D_{it} and $L_{BOND,it}$) compensate the investor for default and liquidity risk:

$$\pi_{it} = D_{it} + L_{BOND,it}. \tag{2}$$

None of the three bond yield components are directly observable. However, investors observe the cost of insurance against default of sovereign i on the corresponding Credit Default Swap (CDS):

$$CDS_{it} = D_{it} + L_{CDS,it}, \tag{3}$$

where $L_{CDS,it}$ is the liquidity risk premium on the CDS.

The CRF rate can be uncovered by extracting the first common factor in the difference between the bond yield and the CDS rate.[2] In particular, a common component (Z_t) with a mean and variance equal to zero and unity, respectively, can be extracted from the CDS-adjusted bond yields by factor analysis (see Jöreskog, 1969) of the model:

$$x_{it} \equiv y_{it} - CDS_{it} = a_i + b_i Z_t + e_{it}, \tag{4}$$

[2] The first factor explains more of the variation in the CDS-adjusted bond yields than any other common factor.

where a_i is a country specific fixed effect and b_i is the factor loading for country i. The disturbances e_{it} are assumed to be independently drawn from a normal distribution with zero mean and variance s_i^2. Rotation of Z_t gives the CRF rate:

$$R_t = \alpha + \beta Z_t, \qquad (5)$$

where α and β are unknown parameters. We allow the risk premiums π_{it} to be dependent on the CRF rate R_t, so that factor loadings can differ across bonds. Note that the liquidity and credit risk premiums may also exhibit significant other common factors that possibly are even more important than the CRF rate for the variation in the risk premiums. However, common factors, if there are any, in the difference in the liquidity risk premium between the bond and the corresponding CDS, are assumed to be less important than the CRF rate for the variation in the bond yields.

In order to fix the multiplicative scaling factor β we assume that one of the bonds in *N* is a benchmark bond (*B*). Benchmark bonds are used to discover market-wide phenomena. In the absence of benchmark bond specific news (such as a deterioration of credit quality) investors attribute price changes of the benchmark bond fully to the risk free rate, implying that the sensitivity of the benchmark bond to the CRF rate equals unity, that is:

$$\beta = b_B. \qquad (6)$$

The risk premium is orthogonal to the CRF rate for bonds with factor loadings as in the right-hand side of Equation (6).

The additive scaling factor α follows by leaving no unexplained fixed components in the deviations of the CDS-adjusted bond yields from the CRF rate, once, these deviations, in turn, are corrected for differences in bond and CDS liquidity risk. To do so, we need some information about liquidity risk. There is no direct information available on future transaction costs. Current transaction costs ($T_{BOND,it}$ and $T_{CDS,it}$) however can be observed. It is reasonable to expect that investors take into account the current transaction cost on the bond and the CDS when pricing possible future transaction cost. For example, in the euro area, the transaction cost on sovereign CDS has been systematically above the transaction cost on sovereign bonds. Investors are then likely to demand a higher liquidity risk premium on the CDS than on the bond. We exploit this information by assuming that the liquidity risk premium is positively and linearly related to current transaction cost,

$$L_{BOND,it} = T_{BOND,it}\gamma \text{ and } L_{CDS,it} = T_{CDS,it}\gamma\,, \tag{7}$$

where γ is an unknown parameter. We assume that the liquidity risk premiums for the bond and the CDS depend in the same way on the respective transaction costs as there are no obvious arguments for the opposite. Both α and γ can now be estimated by ordinary regression of the model:

$$x_{it} - b_B\hat{Z}_t = \alpha + (T_{BOND,it} - T_{CDS,it})\gamma + \xi_{it}\,, \tag{8}$$

where $\hat{Z}_t$ is the derived common component and ξ_{it} is an error term. Finally, the credit risk premium immediately follows once the CRF rate and liquidity risk premium are known.

3 Improving the efficiency of factor analysis for one-factor models

A new technique is proposed for the factor analysis of equation (4), which is explained in detail in this section. Throughout we assume that the only available information about the exogenous process stems from the CDS-adjusted bond yields. We derive cross-variable restrictions, and maximize the likelihood function under these restrictions. Classical factor analysis (see Jöreskog's, 1969) ignores these restrictions, leading to parameter estimates that are possibly infeasible because of their implications (*i.e.* negative variances). Since our method (called Longitudinal Factor Analysis) exploits the typically very large longitudinal dimension of bond data, it performs better than traditional factor analysis methods that rely on the much smaller cross-sectional dimension. The core of the estimation method is to regress bonds on each other. The original model parameters can then be extracted from the estimation results of these inter-bond regressions. For three bonds the model parameters are uniquely pinned down, so the discussion of the method starts with this special case.

3.1 Estimating with three bonds

We can remove the dependency of bond *i* on the exogenous factor by regressing on a different bond *j*. Using the definition of x_{it} and x_{jt} from Equation (4) we can write this regression in terms of the original model parameters as:

$$x_{it} = a_i + b_i Z_t + e_{it} = a_i + b_i\left(\frac{x_{jt}}{b_j} - \frac{a_j}{b_j} - \frac{e_{jt}}{b_j}\right) + e_{it} =$$

$$a_i - \frac{b_i}{b_j} a_j + \frac{b_i}{b_j} x_{jt} + e_{it} - \frac{b_i}{b_j} e_{jt} . \qquad (9)$$

As Z_t drops out from the equation, at this point no assumption needs to be made about its distribution. Equation (9) shows that regressing bond *i* on bond *j* yields constant a_i – $(b_i/b_j)a_j$ and coefficient b_i/b_j. Note that alternatively we could have regressed bond *j* on bond *i*. This would have led to $x_{jt} = a_j - (b_j/b_i)a_i + (b_j/b_i)x_{it} + e_{jt} - (b_j/b_i)e_{it}$, which is identical to Equation (9) scaled by b_j/b_i. Hence, all information can be obtained from either of these regressions.

Equation (9) further shows that s_{ij}^2, the variance of the disturbances when regressing bond *i* on bond *j*, satisfies:

$$s_{ij}^2 = s_i^2 + \left(\frac{b_i}{b_j}\right)^2 s_j^2 . \qquad (10)$$

Although b_i / b_j is known since it is the coefficient of the inter-bond regression, s_i^2 and s_j^2 are not pinned down since we have only one equation and two unknowns. Two bonds are thus not sufficient to extract the parameters of the original model.

However, now consider the case where we have three bonds. Let *i*, *j*, *k* denote different bonds and regress bond *i* on bond *j*, *i* on *k* and *j* on *k*. The variances of the disturbances give the following system:

$$\begin{cases} s_{ij}^2 = s_i^2 + \left(\frac{b_i}{b_j}\right)^2 s_j^2 \\ s_{ik}^2 = s_i^2 + \left(\frac{b_i}{b_k}\right)^2 s_k^2 \\ s_{jk}^2 = s_j^2 + \left(\frac{b_j}{b_k}\right)^2 s_k^2 \end{cases} . \qquad (11)$$

Solving the three equations for s_i^2, s_j^2 and s_k^2 yields:

$$\begin{cases} s_i^2 = \frac{1}{2}\left(s_{ij}^2 + s_{ik}^2 - \left(\frac{b_i}{b_j}\right)^2 s_{jk}^2 \right) \\ s_j^2 = \frac{1}{2}\left(\left(\frac{b_j}{b_i}\right)^2 s_{ij}^2 + s_{jk}^2 - \left(\frac{b_j}{b_i}\right)^2 s_{ik}^2 \right) \\ s_k^2 = \frac{1}{2}\left(\left(\frac{b_k}{b_i}\right)^2 s_{ik}^2 + \left(\frac{b_k}{b_j}\right)^2 s_{jk}^2 - \left(\frac{b_k}{b_i}\right)^2 s_{ij}^2 \right) \end{cases} . \qquad (12)$$

The variances of the bonds can now be obtained after first computing the errors of the inter-bond regressions according to Equation (9) and then applying System (12) with the resulting variances.

Equation (9) shows that the errors of the three inter-bond regressions are not independent. Similarly, when the ratios b_i/b_j and b_i/b_k are known, the ratio b_j/b_k follows. Hence, we cannot perform these regressions independently. Instead, we obtain estimates of the constants and factor loadings by maximizing the likelihood of the errors in Equation (4). In this way, we can exploit that the errors e_{it}, e_{jt} and e_{kt} are independent by assumption. Standard maximum likelihood procedures can be used. Since we assume that all our information about the exogenous process stems from the bonds, we do not want to impose a particular distribution on Z_t. We thus treat the exogenous factor as having an improper uniform density function, which is constant on the real line and thus uninformative. The infinite mass does not cause problems when conditioning on Z_t (see Hartigan, 1983). Up to the constant density of Z_t, the likelihood of observing the errors is thus:

$$\prod_{t=1}^{T} \frac{1}{(2\pi)^{3/2} s_i s_j s_k} \int_{Z_t=-\infty}^{\infty} e^{-\frac{1}{2}\frac{(x_{it}-a_i-b_iZ_t)^2}{s_i^2}} e^{-\frac{1}{2}\frac{(x_{jt}-a_j-b_jZ_t)^2}{s_j^2}} e^{-\frac{1}{2}\frac{(x_{kt}-a_k-b_kZ_t)^2}{s_k^2}} dZ_t . \qquad (13)$$

Under the restrictions of System (12), Equation (13) is maximized over the constants a (*i.e.* a_i, a_j and a_k) and factor loadings b (*i.e.* b_i, b_j and b_k).[3]

Restrictions on the exogenous process are needed to obtain the individual factor loadings and constants. We can think of the factor loadings as measuring how sensitive the bonds are to the volatility of the underlying factor. For given factor loadings, the constants a pin down the level of the bonds. Since both the level of the exogenous process and its variability are unknown, there are two degrees of freedom and we thus need two restrictions.[4] To emphasize, given the bond yields, restrictions on the level and volatility of the exogenous process affect a and b, and vice versa. We assume that the level and variability of the exogenous process are equal to 0 and 1 respectively.

[3] For practical purposes the computation of the T integrals is not convenient. By recognizing from the integrand that Z_t conditioned on x_{it}, x_{jt} and x_{kt} has a normal density, each integral can be replaced. See Appendix I for the details.

[4] Standardizing only the level of the exogenous process to C is not sufficient. To see this, note that if the level of a particular exogenous process Z satisfies this condition, so does the process $uZ + (1 - u)C$. Hence, the standardization does not only need to fix the level of the exogenous process, but also its variability.

To make the standardization of Z operational, we need expressions for the expectation and variability of the exogenous process. The maximum likelihood estimate of the exogenous process $\hat{Z}$ is derived in Appendix I. Although we assume the most uninformative distribution of the exogenous factor, conditional on the three bonds it has a normal distribution with conditional expectation $\hat{Z}_t$ given by:

$$\hat{Z}_t = \frac{\frac{b_i^2}{s_i^2}\frac{x_{it}-a_i}{b_i} + \frac{b_j^2}{s_j^2}\frac{x_{jt}-a_j}{b_j} + \frac{b_k^2}{s_k^2}\frac{x_{kt}-a_k}{b_k}}{\frac{b_i^2}{s_i^2} + \frac{b_j^2}{s_j^2} + \frac{b_k^2}{s_k^2}}, \quad (14)$$

which is an unbiased estimate of Z_t with estimation variance $s_{\hat{Z}}^2 = (b_i^2 / s_i^2 + b_j^2 / s_j^2 + b_k^2 / s_k^2)^{-1}$. Note that the individual bonds are first centered by a and scaled by b since $(x_{it} - a_i)/b_i$ is the best estimate of the underlying process conditional on x_{it} (see Equation (4)). Then a weighted average is taken over the three scaled bonds with the relative weights being the squared factor loadings over the variances. Hence, relative weights are high if bonds have a low idiosyncratic risk compared to their dependence on the exogenous factor. Intuitively, these bonds are good predictors of the exogenous factor. In fact, the weights b_i^2 / s_i^2 are the inverse of the variance of the predictions $(x_{it} - a_i)/b_i$. $\hat{Z}_t$ is the best linear unbiased estimate of Z_t.

While it is too complicated to obtain a full analytical solution of the maximization problem, our standardization makes it possible to find analytical expressions for the estimator $\hat{a}$ of the constants. In Appendix I it is shown that the maximum likelihood solution satisfies:

$$\frac{\overline{E}_t[x_{it}] - a_i}{b_i} = \frac{\overline{E}_t[x_{jt}] - a_j}{b_j} = \frac{\overline{E}_t[x_{kt}] - a_k}{b_k}, \quad (15)$$

where the operator $\overline{E}_t$ denotes the time-average in the sample, *i.e.* $\overline{E}_t[x_{it}] = \frac{1}{T}\sum_{t=1}^{T} x_{it}$.

When now taking time-averages of both sides of Equation (14) and using the above equalities, we find the following unbiased estimate for the level of the exogenous process:

$$\bar{E}_t\left[\hat{Z}_t\right] = \frac{\frac{b_i^2}{s_i^2}\frac{\bar{E}_t[x_{it}]-a_i}{b_i} + \frac{b_j^2}{s_j^2}\frac{\bar{E}_t\left[x_{jt}\right]-a_j}{b_j} + \frac{b_k^2}{s_k^2}\frac{\bar{E}_t[x_{kt}]-a_k}{b_k}}{\frac{b_i^2}{s_i^2} + \frac{b_j^2}{s_j^2} + \frac{b_k^2}{s_k^2}} = \frac{\bar{E}_t[x_{it}]-a_i}{b_i}. \quad (16)$$

When the level of the exogenous process is standardized to 0, this equation is not dependent on b_i anymore and it directly follows that $\hat{a}_i = \bar{E}_t[x_{it}]$. Similarly, $\hat{a}_j = \bar{E}_t\left[x_{jt}\right]$ and $\hat{a}_k = \bar{E}_t[x_{kt}]$.

To obtain estimates $\hat{b}$ for the factor loadings, we need to standardize the variability of the exogenous process. The variance of the estimate of Z_t (V[$\hat{Z}_t$]) equals the sum of the variance of Z_t (V[Z_t]) and the variance of the estimation error ($s_{\hat{Z}}^2$). Hence we can estimate the variance of the exogenous process by:

$$V[Z_t] = V\left[\hat{Z}_t\right] - s_{\hat{Z}}^2 = \frac{1}{b_i^2}\left(\frac{V\left[\frac{1}{s_i^2}x_{it} + \frac{b_j}{b_i}\frac{1}{s_j^2}x_{jt} + \frac{b_k}{b_i}\frac{1}{s_k^2}x_{kt}\right]}{\left(\frac{1}{s_i^2} + \frac{b_j^2}{b_i^2}\frac{1}{s_j^2} + \frac{b_k^2}{b_i^2}\frac{1}{s_k^2}\right)^2} - \frac{1}{\frac{1}{s_i^2} + \frac{b_j^2}{b_i^2}\frac{1}{s_j^2} + \frac{b_k^2}{b_i^2}\frac{1}{s_k^2}}\right). \quad (17)$$

When the ratios b_i / b_j and b_i / b_k and the variances of the disturbances are known, imposing the variance of the exogenous process to be 1 pins down $\hat{b}_i$ and hence the estimates for the other factor loadings. Thus, we can maximize Equation (13) over two ratios of factor loadings, say b_i/b_j and b_i/b_k, since the constants are known, and the individual factor loadings follow from the standardization of Equation (17).

3.2 *Estimating with more than three bonds*

In case of n bonds there are $n(n-1)/2$ possible pairs for inter-bond regressions and n variances to be found. The analogue of System (12) will be a system of $n(n-1)/2$ equations in n unknowns. For $n > 3$ there will be more equations than unknowns. Although in theory (*i.e.* asymptotically) solving any n equations that pin down all unknowns would yield variances that solve the other equations as well, for finite samples this will not be the case. Hence, some cross-variable restrictions would possibly be violated and there is no obvious way to decide which restrictions can be ignored.

We circumvent this problem by exploiting the fact that for three bonds all cross-variable restrictions can be imposed and apply the method therefore to each possible set of three bonds. For n bonds, we have $n(n - 1)(n - 2)/6$ of these sets. Each of these sets provides an estimate for the parameters of the included bonds. Let H_i contain all $(n - 1)(n - 2)/2$ sets of three bonds that include bond i. It follows that for each bond there are $(n - 1)(n - 2)/2$ estimates for the corresponding factor loading and error variance. Although some of the cross-variable restrictions may still be violated, perhaps the most efficient way to obtain the system parameters is to compute the average of the respective estimates over all possible sets of three bonds. Note that this is only possible because the standardization of the external process leads to similarly scaled factor loadings.

By following this strategy the main steps of LFA can be summarized as follows.

3.3 Summary of LFA

1) For each set $h \in H_i$ of three bonds including bond i the likelihood function in Equation (13) is maximized over b_i / b_j and b_i / b_k (using the representation of Equation (21) in Appendix I) under the restrictions of System (12).
2) The estimate of the constant for bond i is $\hat{a}_i = \frac{1}{T}\sum_{t=1}^{T} x_{it}$.
3) An intermediate estimate ($\hat{b}_{i,h}$) of the factor loading b_i for each set $h \in H_i$ is obtained from Equation (17) with the standardization V[Z_t] = 1.
4) An intermediate estimate ($\hat{s}_{i,h}^2$) of the error variance s_i^2 for each set $h \in H_i$ is obtained from System (12) after computing the error variances s_{ij}^2, s_{ik}^2 and s_{jk}^2 of the inter-bond regressions.
5) Final estimates are computed by averaging over all possible sets of three bonds: $\hat{b}_i = \frac{2}{(n-1)(n-2)}\sum_{h \in H_i}\hat{b}_{i,h}$ and $\hat{s}_i^2 = \frac{2}{(n-1)(n-2)}\sum_{h \in H_i}\hat{s}_{i,h}^2$.
6) The final estimate of the external process Z is obtained by extending Equation (14) to include all bonds.

3.4 A Monte Carlo experiment

To asses the performance of LFA we have run simulations to compare the results under our method with those obtained by Principal Components (PC) based on Theil's (1971) method and factor-analysis based on the EM algorithm of Rubin and Thayer (1982). In line with the empirical Section 4 on the CRF rate for the euro area the experiment considers twelve bonds with different sensitivities to the common factor and risks. Bond yield series of one thousand observations each are generated according to Equation (4) without constant and with factor loadings equal to $b_i = 1 + (i-1)/10$,

$i \in \{1,...,12\}$. The disturbances corresponding to bond i are drawn from a normal distribution with zero mean and variance equal to $s_i^2 = (0.2 + (i-1)/10)^2$, $i \in \{1,...,12\}$. The common component Z_t is drawn from a standard normal distribution. To be consistent with the model in Section 2, idiosyncratic risk is measured by s_i^2 / b_i^2. Under these definitions the factor loadings increase from $b_1 = 1$ to $b_{12} = 2.1$ and the idiosyncratic risks increase from $s_1^2 / b_1^2 = 0.04$ to approximately $s_{12}^2 / b_{12}^2 = 0.38$.

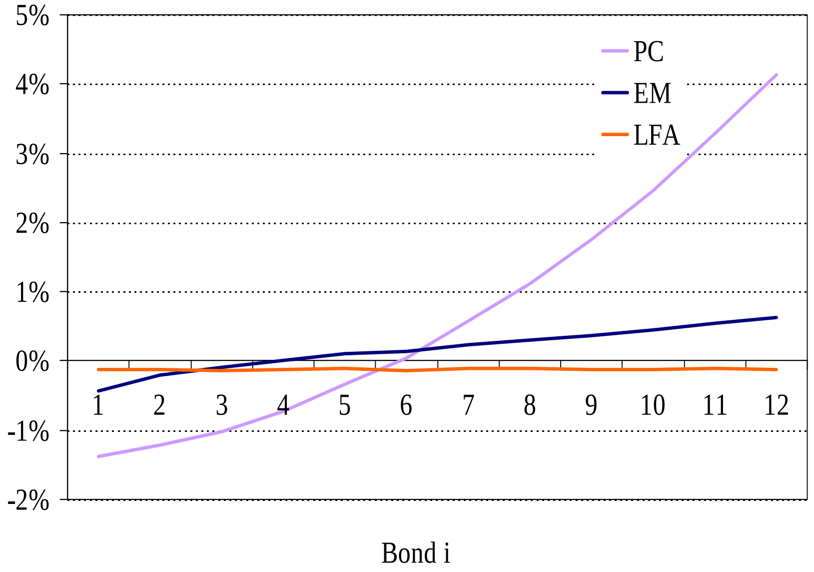
Average estimates are computed over ten-thousand replications. Table 7 of Appendix II reports the average factor loadings and average variances of the errors. Figure 1a shows the average error in the factor loading (E[$\hat{b}_i$]/b_i -1) whereas Figure 1b shows the average error in the idiosyncratic risk (E[$\hat{s}_i^2 \big/ \hat{b}_i^2$]/($s_i^2 / b_i^2$) -1) of bond i. Figure 1c shows the average absolute error in the estimated common component, which is calculated as $E\left[\frac{1}{T}\sum_{t=1}^{T}\left|\hat{Z}_t - Z_t\right|\right]$.

LFA is better in predicting the factor loading and idiosyncratic risk of most bonds. Both PC and EM underestimate (overestimate) the factor loading of relatively low (high) risk bonds up to 4% and 0.6% of the true value respectively. LFA underestimates the factor loading of all bonds up to only 0.1% of the true value (see Figure 1a). For some bonds the estimation error in the factor loading is 82% lower for LFA than for EM, and the efficiency gain is even larger when LFA is compared to PC. Moreover, PC and EM substantially overestimate (underestimate) the idiosyncratic risk on relatively low (high) risk bonds. The deviation is up to 73% and 27% of the true risk for the two respective methods. In sharp contrast, LFA overestimates the idiosyncratic risk of all bonds only up to 0.4% of the true risk (see Figure 1b). LFA is superior to PC in the estimation of the idiosyncratic risk of all bonds. EM has a slightly smaller estimation error than LFA in the idiosyncratic risk of bond *i=5*. However, in all other cases LFA is superior to EM in the estimation of the idiosyncratic risk. Clearly, exploiting the longitudinal dimension of the sample indeed leads to overall better estimates of the idiosyncratic risk.

LFA is also better in predicting the common component. The estimation error in Z_t is on average 34% smaller for LFA than PC and 2% smaller for LFA than EM (see Figure 1c). Although the latter gain is modest, one should keep in mind that the factor loading of the benchmark bond is needed to compute the CRF rate. The efficiency gain from using LFA instead of EM when estimating the CRF rate thus comes first and foremost from an improvement in the estimation of the factor loadings.

In sum, LFA clearly outperforms PC and classical factor analysis when the cross-sectional dimension of the panel data set is small and the longitudinal dimension is large. Note, however, that so far our estimation technique is limited to models with one common factor only.[5]

Figure 1a: Average error in the estimated factor loadings



[5] In the case of two common factors, at least four bonds are needed to identify the system parameters. Using four bonds yields a system of four variance equations (restrictions) in four unknowns. However, the system has rank three and does not have a unique solution.

Figure 1b: Average error in the estimated idiosyncratic risks

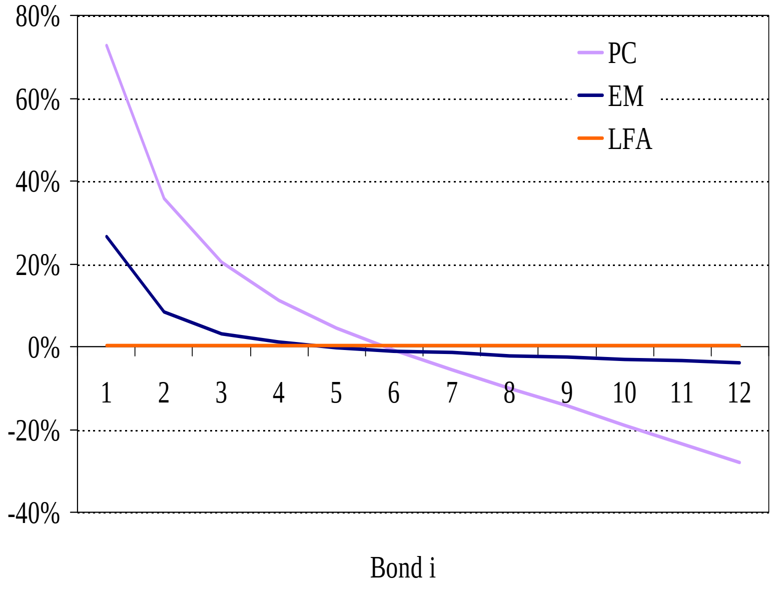
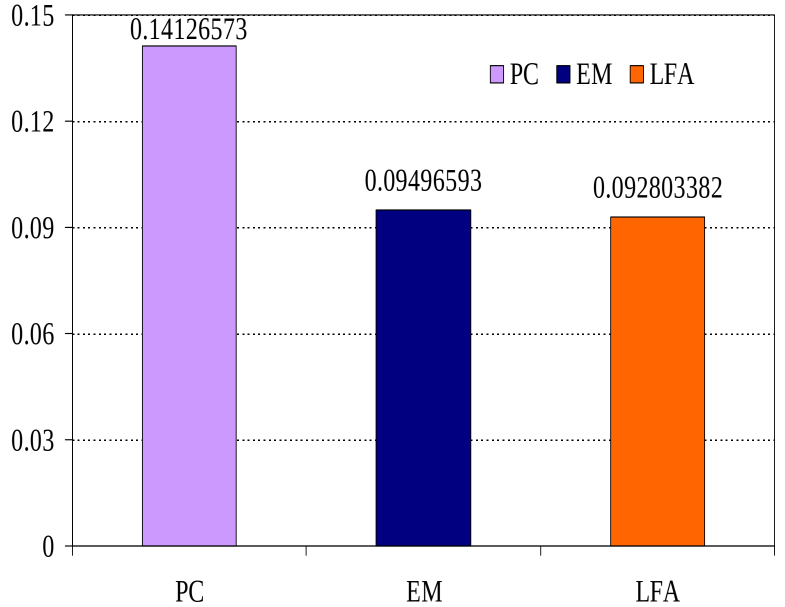


Figure 1c: Average absolute error in the estimated common component



Note: See the notes of Appendix II for details of the simulation.

4 The common risk free rate in the European Monetary Union

4.1 The data

We analyze bonds of eleven countries that are part of the European Monetary Union: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain, and bonds of the European Investment Bank (EIB).[6] The period under analysis is Feb. 2006 till Feb. 2010. Using individual plain-vanilla bond quotes on Bloomberg, 5-year and 10-year spot yields are constructed for each country (see Appendix III).[7] One caveat is worth mentioning upfront. In normal times bond quotes are close to transaction prices. However, in times of financial market stress there can be important discrepancies.

Where available, the on-the-run bond (*i.e.* the most recent issue in the reference maturity) is used when inter- or extrapolating points on the yield curve. Blanco (2001) finds that on-the-run French, German and Spanish sovereign bonds have significantly lower yields than off-the-run bonds. For example, between January 1999 and May 2001, the yield on on-the-run German bonds was about six b.p. lower than on off-the-run bonds. On-the-run bonds are thus closer to the risk free rate, and therefore of particular interest for this study. Ejsing and Sihvonen (2009) find that, between January 2006 and September 2008, on-the-run status has only a modest effect on the pricing of German sovereign bonds when other factors have not been controlled for. On-the-run status however is found to have a significant positive impact on liquidity, even after controlling for substantial spillover effects from Bund future contracts to cash bonds, which in turn has a negative impact on the yield.[8] In contrast with our bond selection strategy, Gürkaynak et al. (2006) exclude on-the-run bonds and their deputies (*i.e.* the second most recently issued bond) when estimating the US treasury yield curve so that the liquidity of the included securities is relatively uniform. In addition to liquidity effects, Pasquariello and Vega (2007) find also other factors, such as bond maturity, that can explain yield differences between on-the-run and off-the-run US treasury bonds. Exclusion of on-the-run bonds could thus bias the estimate of the risk free rate.

[6] Cyprus, Luxembourg and Malta are excluded due to their size; members of EMU that only recently joined such as Slovenia and Slovakia are excluded to have a longer time-span for the older members.
[7] Bloomberg reports *z*-spreads only since Feb. 2006.
[8] Table 3, 4 and 5 of Ejsing and Sihvonen (2009) show that on-the-run status increases the trading volumes and quoted depth (*i.e.* volume available for trading at the best three bid and offer prices) and lowers the bid-ask spread, respectively. Table 9 shows that liquidity, when measured by the bid-ask spread, can explain part of the differences between French and German bond yields.

Other bond selection criteria are discussed in Appendix III. For instance, French, German and Italian bonds are required to have a minimum size of € 1bn, for all other countries and the EIB € 500mn is the minimum size. The number of outstanding bonds and corresponding face value are substantially higher for France, Germany, and Italy in comparison to the other countries (see Table 1). Between Feb. 2006 and Feb. 2010, the bonds of these three countries together covered almost half the number of total bonds and two-thirds of the total amount outstanding on bonds with an original maturity of more than four years. The total size of the sovereign plain-vanilla long-term (large-sized) bond market was about € 3.5tr (see Table 1).

Table 1: The size of the long-term sovereign bond market in the euro area between Feb. 2006 and Feb. 2010

	Average number	Average amount outstanding (in € million)
Austria	17	134
Belgium	20	225
Finland	9	46
France	43	729
Germany	42	750
Greece	22	168
Ireland	10	52
Italy	38	722
Netherlands	18	187
Portugal	15	83
Spain	23	271
EIB	18	81
Total	274	3448

Note: Only large fixed rate plain vanilla bonds with an original maturity of more than four years are included.

Table 8 of Appendix IV provides statistics about the bonds used in the estimation of the spot yields by reference maturity. The number of bonds used varies from five in the case of 5-year Irish bonds to twenty in the case of 5-year German bonds. All countries and the EIB have issued at least one on–the-run bond in both the 5-year and 10-year reference maturity during or just before the sample period since the minimum original maturity of the bonds used is within the reference maturity interval (see Appendix III for a definition). The maximum original maturity is in all cases at least five years above the reference maturity. For example, the maximum original maturity of Austrian bonds that are potentially used to estimate the 10-year CRF is 15.5 years, i.e. 5.5 years higher than the reference maturity. Note that the remaining time to maturity of these bonds is usually close to the reference maturity on the dates when they are used for interpolation. Except for 5-year EIB bonds, the minimum size of the bonds used to calculate the spot yields is substantially higher than the minimum size selection criterion mentioned above. On average, used bonds have a face value of at least € 5 bn.

The 5-year and 10-year bond spot yields are shown in Figure 2a and 2b respectively. Two things are worth pointing out. 1) until spring 2007 the lowest yields are found on Finnish, German and Irish bonds. Throughout the sample period the highest yields are usually found on Greek bonds except for an intermediate period between May 2008 and October 2009 when Irish bonds earn the highest yields. As a result of the financial crisis, Irish bonds thus went from the most expensive to the cheapest bonds in the euro area. 2) while yields were moving nearly synchronously until the first half of 2007, the cross-country variance in bond yields increased massively during the financial crisis. The difference between minimum and maximum yields rose from about 30 b.p. in 2006 to more than 400 b.p. in 2010. Basic descriptive statistics of the bond spot rates are shown in Table 9 of Appendix IV.

Fig. 2a: 5-year bond spot yields (in %)

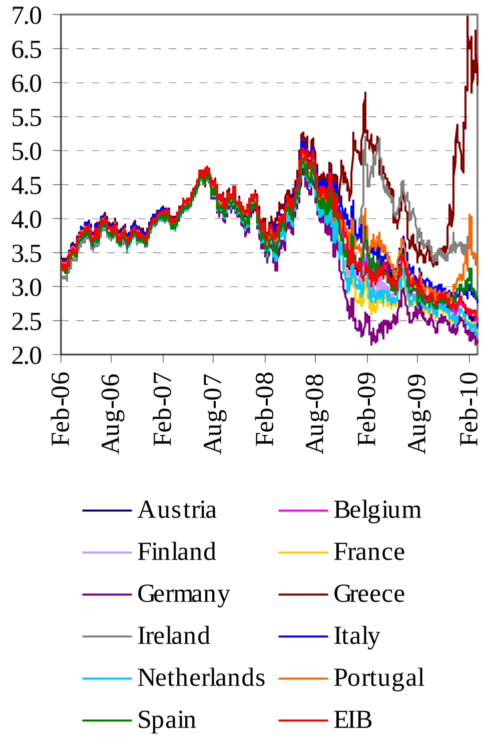
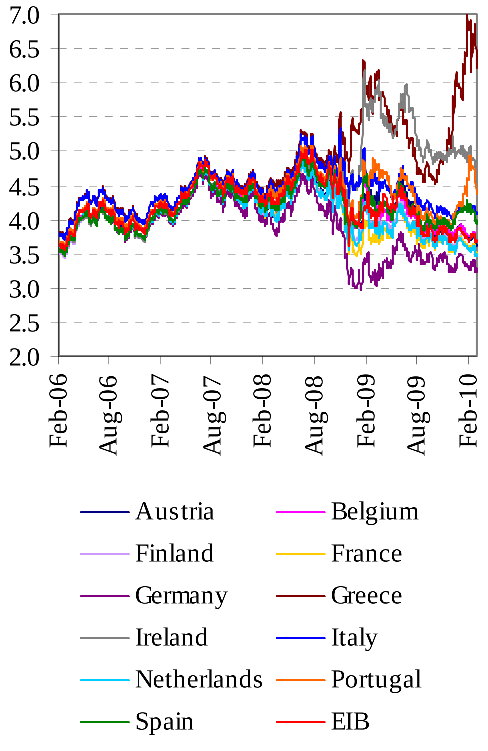


Fig. 2b: 10-year bond spot yields (in %)



Data on sovereign and EIB Credit Default Swap (CDS) quotes are from Credit Market Analysis Limited and Markit respectively (see Table 10 of Appendix IV for descriptive statistics). The EIB CDS data must be cautiously interpreted as the reported quotes may not necessarily reflect true trading opportunities. To the best of the authors' knowledge, so far EIB CDS have actually never been traded.

Figure 3 shows the CDS spread evolution over the sample period by reference maturity. There are again two points worth mentioning. 1) before the crisis the lowest CDS spreads are observed for Austria, France and the Netherlands. Recall that during this period Finish, German and Irish bonds earned the lowest yields. Note however that there are no pre-crisis data on Finnish CDS spreads available. Since the crisis, CDS spreads are among the lowest for Finland, Germany, and the EIB. The cost of insurance against a default of the EIB is only marginally higher, or sometimes even below, the insurance cost on Finnish or German debt. The CDS spreads are the highest for Ireland, Italy and Greece. 2) by comparing the CDS spreads in Figure 3 with the bond yields in Figure 2 we observe that until the second half of 2007 changes in bond spot yields were not driven by changes in CDS spreads. Indeed, between Feb. 2006 and Aug. 2007, bond spot yields rose by more than 100 b.p. while CDS spreads were basically flat. In contrast, since the summer of 2007, a large part of the variation in bond yields is caused by swings in the credit risk premiums. In the next two sections we will determine what role the CRF rate and liquidity premiums have played.

Fig. 3a: 5-year CDS spreads (in b.p.)

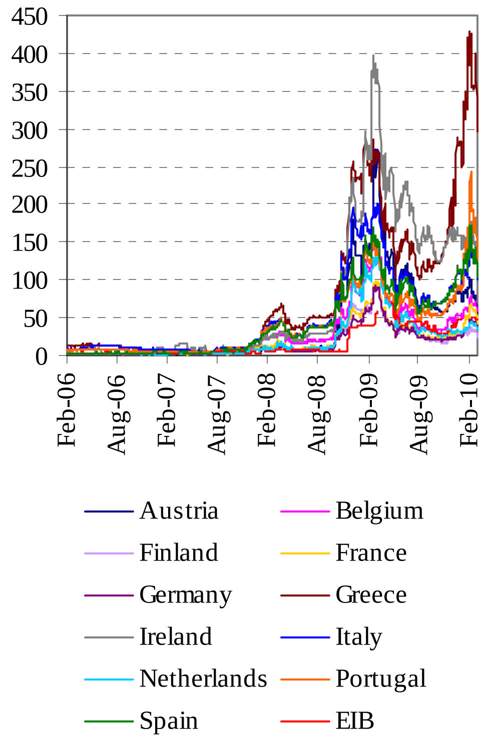
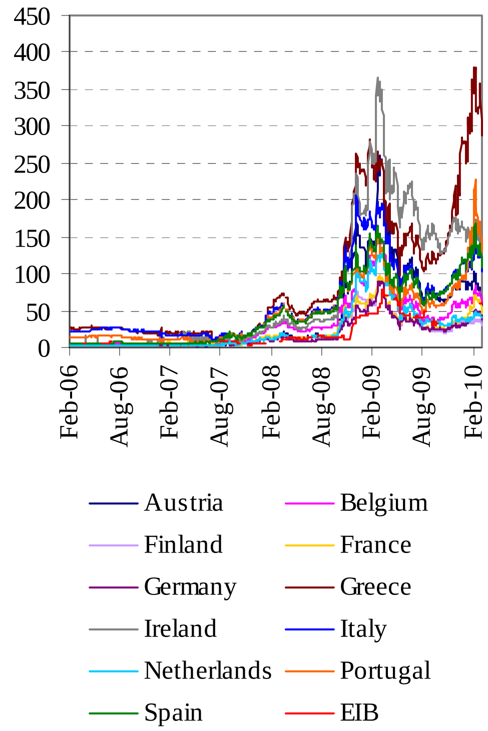


Fig. 3b: 10-year CDS spreads (in b.p.)



We measure transaction cost by the respective bid-ask spread on the bond and the CDS. The difference between the bid and the ask rate is the most important cost incurred by the investor when buying and subsequently selling a security. Table 11 of Appendix IV shows the average transaction cost by reference maturity and sub-sample period. The sample is broken up in a pre-crisis period from Feb. 2006 until Jun. 2007 included, and a financial crisis period from July 2007 until the end of our sample, *i.e.* Feb. 2010. Although sovereign bond markets were initially little affected, the beginning of the financial crisis is thus associated with the sub-prime mortgage crisis in the US. The main features of the transaction cost data are as follows. First, in all cases, the transaction cost on the CDS is higher than on the corresponding bond. Before the outbreak of the recent financial crisis, the 5-year bond transaction cost was on average about 0.6 b.p. whereas the five-year CDS transaction costs was on average about 1.6 b.p.. For 10-year bonds and CDS, the average transaction cost was 0.4 b.p. and 2.6 b.p. respectively. Note that the 10-year bond transaction cost is lower than the 5-year bond transaction cost whereas the 10-year CDS transaction cost is higher than the 5-year CDS transaction cost. Hence, the impact of maturity on trading cost is ambiguous. Secondly, in almost all cases, the CDS transaction cost increased more than the bond transaction cost during the financial crisis. France, Italy and Portugal are exceptions as for these countries the transaction cost on 10-year CDS increased less fast than the transaction cost on the corresponding bond. Thirdly, the pre-crisis data do not reveal a clear cross-sectional relationship between the bond and the CDS transaction cost. Since the crisis, however, bonds of countries with relatively low bond transaction cost also tend to have relatively low CDS transaction cost, as shown by the scatter plots in Figure 4 for eleven out of the twelve issuers. Unfortunately, CDS transaction costs are not available for the EIB.

Fig. 4a: Average 5-year bond against CDS transaction cost (in b.p.)

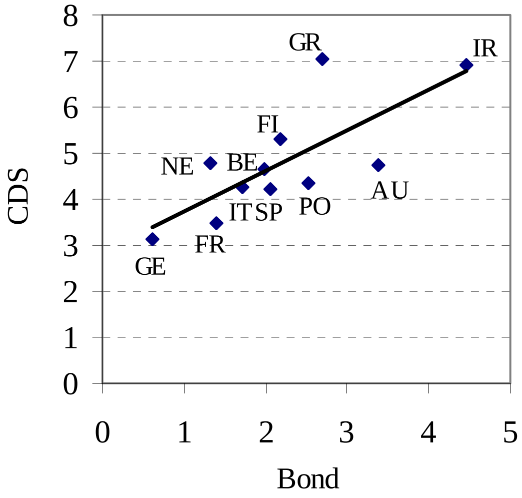
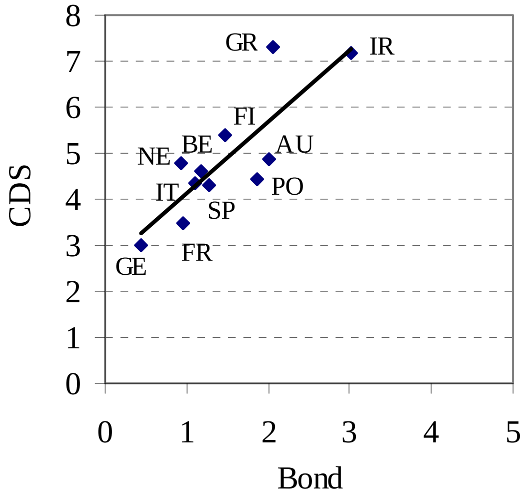


Fig. 4b: Average 10-year bond against CDS transaction cost (in b.p.)



Note: The transaction cost is measured by the difference in the bid and the ask rate. The average value is computed over the financial crisis period Jul. 2007 – Feb. 2010.

The data description points out several issues that seem important for the estimation procedure. Figure 3 raises the question whether the factor loadings in Equation (4) could be unstable because of a regime shift in the importance of the credit risk premiums. Furthermore, pre-crisis Irish bonds seem to be clear outliers since their low bond yields seem inconsistent with their high transaction costs. Last but not least, the transaction cost data suggest that, under the assumptions of the model in Section 2, the CDS-adjusted bond yields should be on average lower than the CRF rate because the right-hand side explanatory variable of Equation (8), *i.e.* the difference in the transaction cost between the bond and the CDS, is on average negative. Preliminary analysis of the data however reveals that for some countries the CDS-adjusted bond yields were on average above the CRF rate, suggesting that the credit risk premium implied on the CDS can differ from the one on the bond.

4.2 The Common Risk Free rate

Taking into account the results of the data inspection, the following strategy is applied when estimating the common risk free rate:

1) Over the pre-crisis (Feb. 2006 – Jun. 2007) period, factor loadings are assumed to be stable, and factor analysis of Equation (4) is performed on a sample of nine out of the twelve issuers. Finnish and EIB bonds are excluded from the factor analysis because there is no or insufficient information available on the CDS spreads during this period; Irish bonds are excluded because they create outlying observations. Irish and EIB factor loadings are estimated in a second step by OLS regression of their CDS-adjusted bond yields on the common component $\hat{Z}_t$ of Equation (4) obtained in the first step (and a constant). In this way, Irish and EIB bonds do not affect the CRF rate, but their factor loadings can still be obtained. Note that factor analysis requires a balanced sample and that missing observations lead to 5-year and 10-year samples with a different number of observations. By including the EIB in the factor analysis the number of observations *T* would be substantially reduced.

2) Over the financial crisis period (Jul. 2007 – Feb. 2010), factor loadings are allowed to vary, and are estimated over rolling windows of 125 trading days. One window corresponds to half a calendar year. The CRF rate over the rolling windows is based on factor analysis of Equation (4) on a sample of only four issuers. We only include Belgium, France, Germany, and the Netherlands because for some windows the factor loadings of the other eight issuers can be relatively far away from the factor loading concomitant the Bund. These factor loadings are subsequently estimated by OLS regression. When performing factor analysis for the rolling windows we thus include fewer countries than before so that possible measurement errors in the loadings of the excluded countries do not affect the CRF rate. Only the last observation of a rolling window is retained. For example, over the first rolling window that begins on 3 January 2007 and ends on 3 July 2007, the first 124 estimated CRF rates are based on the pre-crisis period analysis and only the last CRF estimate of 3 July 2007 is based on the first rolling window analysis.

3) To obtain the additive scaling factor $\hat{\alpha}$ of Equation (5) and to estimate the relationship between current transaction cost and the liquidity risk premium, OLS is applied to Equation (8) over the full sample period in order to exploit the variation in the explanatory variable of Equation (8) to a maximum. This regression is based on Germany only because during the crisis the Bund has substantially lower credit risk than any other bond. By including only the Bund in this regression we limit possible distortions to the CRF rate that could arise from empirical differences in the implicit credit risk premium on a bond and the implicit credit risk premium on the corresponding CDS. In our theoretical model of Section 2 these premiums were assumed to be equal for each country. As will be shown below, this is the case for Germany, but not for most other countries.

Figure 5 depicts the evolution of the factor loadings by reference maturity obtained with Longitudinal Factor Analysis as developed in Section 3. All loadings are scaled by the loading concomitant the Bund. Hence, factor loadings below 1 indicate that a bond has a lower sensitivity to the CRF rate than the Bund, while the CRF rate has a larger effect on bonds with a higher factor loading. Over the first say 125 windows (*i.e.* the last six months of 2007), except for the Irish and EIB factor loadings, all scaled factor loadings remain relatively close to their pre-crisis period estimates that are not far from unity. It thus seems adequate to assume stable factor loadings and to include a maximum of countries when analysing the pre-crisis period. Over this period, the factor loading varies between 0.93 (0.97) and 1.09 (1.11) on 5-year (10-year) bonds as can be seen from the first observation for each issuer in the figure. The highest loading corresponds to the Greek bond. Depending on maturity, the lowest loading corresponds to the Austrian or EIB bond. In most cases, the difference with the Bund loading is less than 0.03. Since the crisis, however, differences between loadings across countries have been much more important. Furthermore, factor loadings have begun to vary substantially over time. For example, the factor loading corresponding to the 10-year Greek bond first fell to zero before reaching a maximum of 2.7. Even for countries with factor loadings relatively close to the factor loading on the Bund, such as Belgium, France and the Netherlands, factor loadings vary substantially over time.

Fig. 5a: 5-year bond factor loadings

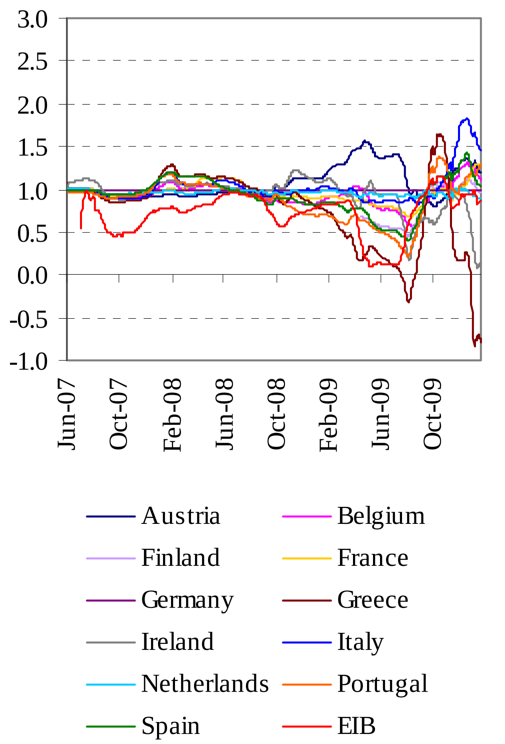
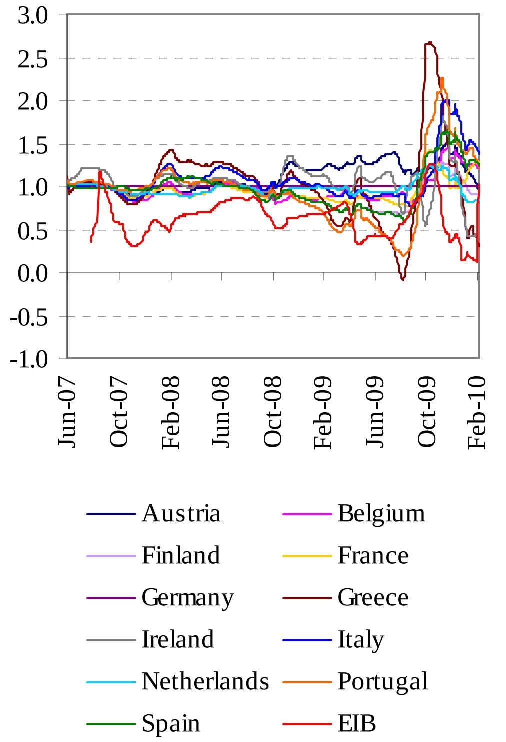


Fig. 5b: 10-year bond factor loadings



Notes: Estimates of the parameters of Equation (4) are obtained with LFA as developed in Section 3. Factor loadings are scaled by the loading concomitant the Bund.

The average of the factor loadings over the rolling windows and the parameter estimates of Equation (8) are shown in Table 2. Note that the cross-sectional differences in average factor loadings are broadly the same for 5-year and 10-year bonds. Between Feb. 2006 and Feb. 2010 the average CRF rate ($\hat{\alpha}$) was respectively 3.32% and 3.71% for 5-year and 10-year bonds, respectively. The coefficient $\hat{\gamma}$ is larger than 1, implying that on average the liquidity risk premium is higher than current transaction cost. Depending on maturity, the current liquidity risk premium is about 1.53 or 1.74 times the current bid-ask spread.

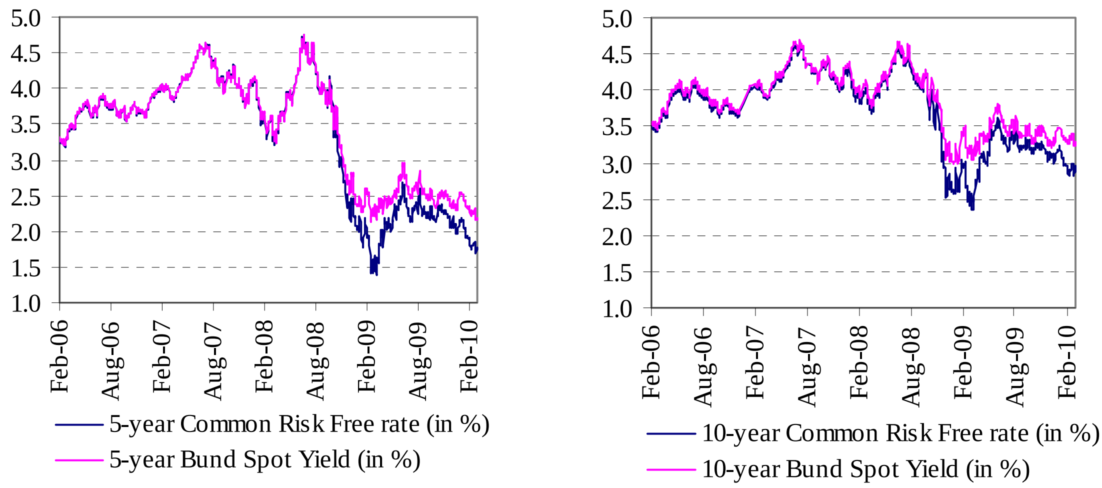
Figures 6a and 6b compare the CRF rate with the Bund yield during the period of analysis. Until the collapse of Lehman Brothers in September 2008, thus more than a year into the sub-prime mortgage crisis, the CRF rate was almost identical to the Bund yield. As will be shown below, between September 2008 and March 2009, default risk, and to some extent liquidity risk, rose substantially on euro area sovereign bonds, the Bund included.

Table 2: LFA estimates of the CRF rate parameters (Feb. 2006 – Feb. 2010)

	Average scaled factor loading over the rolling windows	
	5-year bonds	*10- year bonds*
Austria	1.07	1.11
Belgium	0.94	0.96
Finland	0.80	0.92
France	0.96	0.94
Germany	1.00	1.00
Greece	0.74	1.02
Ireland	0.93	1.04
Italy	1.03	1.08
Netherlands	0.96	0.97
Portugal	0.88	0.96
Spain	0.93	0.98
EIB	0.72	0.63
$\hat{\alpha}$	3.32 (0.03)	3.71 (0.02)
$\hat{\beta}$	0.33	0.26
$\hat{\gamma}$	1.53 (0.09)	1.74 (0.12)
T	1050	1009

Notes: Estimates of the factor loadings are obtained with LFA as developed in Section 3. The standard error is within brackets. Factor loadings are scaled by the loading concomitant the Bund. $\hat{\beta}$ is the average factor loading (before scaling) concomitant the Bund.

Fig. 6a: 5-year CRF and Bund rate (in %) **Fig. 6b: 10-year CRF and Bund rate (in %)**



Note: Based on the LFA estimates of the factor loadings in Table 2.

On average, the 5-year and 10-year CRF rate fell by 76 and 30 b.p., respectively, when the pre-crisis sample period (Feb. 2006 – Jun. 2007) is compared to the crisis sample period (Jul. 2007 – Feb. 2010). Clearly, from peak to bottom level (see Figure 6a and 6b), the 5-year and 10-year CRF rate fell much more (*i.e.* by about 300 and 200 b.p., respectively). Due to the fall in the CRF rate, for many issuers the costs of borrowing were actually lower than before the crisis despite a substantial increase in their risk premiums.

4.3 *The risk premiums*

Next let us show how the risk premiums have behaved in the recent financial crisis. Following the procedure developed in Section 2, the estimated Total Risk Premium (TRP) is decomposed into a Credit Risk Premium (CRP) and a Liquidity Risk Premium (LRP).

The crisis impact on the CRP and LRP are shown in the next four figures. For 5-year bonds (compare Figure 7a with Figure 8a) and 10-year bonds (compare Figure 7b with Figure 8b), the CRP is the dominant component in the TRP. The 10-year Bund CRP rose from an average of less than 5 b.p. before the crisis to about 17 b.p. during the crisis. The CRP on Greek 10-year bonds rose from about 30 b.p. to an average of 140 b.p. The estimated CRP on the 5-year Irish bond was slightly negative over the pre-crisis period, suggesting that some of the very low Irish bond quotes in the secondary market were unreliable indicators of actual pricing conditions.

The increase in the LRP is relatively small in comparison to the increase in the CRP for all bonds. Before the crisis, except for Irish and EIB bonds, there were no substantial differences in the LRP across euro area sovereign bonds. In this respect, EIB bonds are a class apart as they are often held to maturity. During the crisis, the LRP has grown for all issuers but to various degrees. For example, the LRP on the Bund rose on average by less than half of a basis point. In other cases, including the EIB, the LRP rose sometimes by more than four b.p..

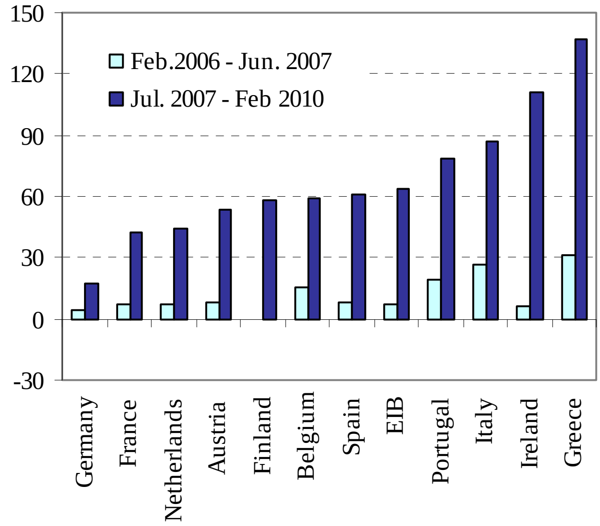
4.4 *Benchmark status, flight-to-quality/liquidity, and CRP comparison*

4.4.1 *Benchmark status*

If, following Dunne et al. (2007), benchmark status is assigned to the bond with the lowest idiosyncratic risk (*i.e.* the standard deviation of the TRP), then the results of Table 3 support our choice to take the Bund as the a priori benchmark. French bonds have the second lowest idiosyncratic risk, but are about twice as risky as the Bund. Greek bonds are the most risky. In addition to the Bund, only French, Dutch and Finnish bonds are less risky than EIB bonds. All other euro area sovereign bonds are more risky than EIB bonds.

Fig. 7a: Average 5-year credit risk premium (in b.p.)

Fig. 7b: Average 10-year credit risk premium (in b.p.)



Note: Based on the LFA estimates of the factor loadings in Table 2. No estimate is available for Finland over the period Feb. 2006 – Jun. 2007.

Fig. 8a: Average 5-year liquidity risk premium (in b.p.)

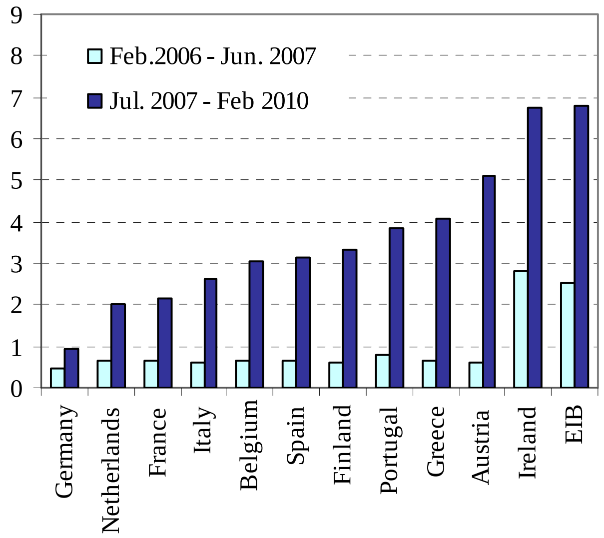
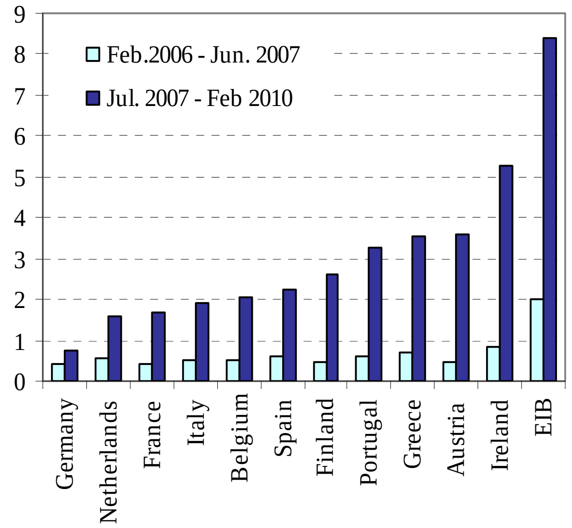


Fig. 8b: Average 10-year liquidity risk premium (in b.p.)



Note: Based on the LFA estimates of the factor loadings in Table 2.

Table 3: Standard deviation of the common risk free (CRF) rate, the credit risk premium (CRP), the liquidity risk premium (LRP), and the total risk premium (TRP), in b.p.

	5-year bonds			*10-year bonds*		
	CRP	LRP	TRP	CRP	LRP	TRP
Austria	36.4	4.5	43.8	39.2	2.6	43.8
Belgium	41.9	1.9	44.5	40.0	1.3	42.2
Finland	34.0	2.4	35.7	29.8	1.8	31.4
France	27.4	1.2	29.6	28.4	1.0	29.4
Germany	17.8	0.4	18.1	14.4	0.3	14.6
Greece	109.9	2.9	115.2	94.8	3.6	102.2
Ireland	82.8	4.9	89.2	88.3	4.0	94.8
Italy	52.6	1.9	52.1	48.6	1.2	50.0
Netherlands	30.8	1.1	33.3	30.4	0.8	33.0
Portugal	51.8	3.2	56.7	49.7	2.5	53.8
Spain	44.3	2.2	47.0	43.7	1.4	45.9
EIB	35.0	3.5	36.6	36.5	4.3	41.3
CRF		88.9			53.6	

Notes: Based on the LFA estimates of the factor loadings shown in Table 2. Sample period: Feb. 2006 – Feb. 2010.

Dunne et al. (2007) designate, with the exception of very long bonds (*i.e.* maturity exceeds 10 years), French bonds as sovereign benchmarks for the period April 2003 – March 2005 because bi-lateral inter-bond regressions suggest that French bonds have lower variance in the total risk premium than Italian or German bonds. Our empirical results are in accordance with this finding for the pre-crisis period up to June 2007 included, but not thereafter. Moreover, before the crisis, there are other bonds, such as Dutch 5-year bonds that are even less risky than French 5-year bonds. Since the crisis, however, Bunds have the lowest risk. Benchmark status shows up in crisis periods.

A robustness check is carried out by taking alternatively any other bond as the *a priori* benchmark. The top row of Table 4 indicates the country on which the estimate of the benchmark choice parameter β is based. The standard deviation of the TRP for each country is shown in the associated column. For all choices, the Bund clearly has the lowest idiosyncratic risk. Hence, the data confirm its benchmark status.

Table 4: Standard deviation of the TRP by benchmark choice parameter β

	A priori benchmark											
	AU	BE	FI	FR	GE	GR	IR	IT	NE	PO	SP	EIB
	5-year bonds											
AU	55	33	n.a.	36	44	32	54	45	41	26	35	n.a.
BE	55	34	n.a.	37	44	33	54	45	41	29	37	n.a.
FI	45	27	n.a.	31	36	31	45	36	34	22	27	n.a.
FR	41	19	n.a.	22	30	19	40	31	26	14	22	n.a.
GE	**30**	**9**	n.a.	**12**	**18**	**11**	**29**	**21**	**15**	**8**	**11**	n.a.
GR	126	106	n.a.	109	115	98	124	118	112	100	108	n.a.
IR	101	78	n.a.	81	89	75	99	91	86	70	81	n.a.
IT	62	41	n.a.	44	52	40	62	53	49	36	44	n.a.
NE	44	22	n.a.	26	33	23	44	35	30	17	25	n.a.
PO	68	47	n.a.	50	57	42	66	59	54	41	49	n.a.
SP	58	37	n.a.	40	47	33	57	49	44	31	39	n.a.
EIB	46	26	n.a.	30	37	29	47	37	34	23	28	n.a.
	AU	BE	FI	FR	GE	GR	IR	IT	NE	PO	SP	EIB
	10-year bonds											
AU	53	36	n.a.	37	44	44	53	46	41	37	39	n.a.
BE	51	34	n.a.	36	42	43	51	45	39	36	37	n.a.
FI	42	25	n.a.	26	31	32	42	34	30	24	26	n.a.
FR	38	22	n.a.	23	29	31	38	32	26	23	25	n.a.
GE	**24**	**9**	n.a.	**9**	**15**	**17**	**24**	**18**	**12**	**11**	**11**	n.a.
GR	111	95	n.a.	96	102	104	111	106	100	98	99	n.a.
IR	104	87	n.a.	88	95	96	104	98	92	88	90	n.a.
IT	59	42	n.a.	43	50	51	59	53	47	43	45	n.a.
NE	42	25	n.a.	26	33	34	42	36	30	26	28	n.a.
PO	63	47	n.a.	48	54	54	62	57	51	48	49	n.a.
SP	55	38	n.a.	39	46	47	55	49	43	40	42	n.a.
EIB	52	32	n.a.	34	41	41	53	43	38	31	34	n.a.

Notes: Based on the LFA estimates of the factor loadings in shown Table 2. The cell with the lowest variance is in bold. Sample period: Feb. 2006 – Feb. 2010.

4.4.2 Flight-to-quality and flight-to-liquidity

While not contesting that credit risk is more important than liquidity risk for the absolute level of the sovereign bond yields in the euro area, Beber et al. (2009) provide some evidence for the hypothesis that, in times of market stress, investors chase liquidity, and not credit quality. The authors argue that large bond trades are almost exclusively driven by liquidity since liquidity has a positive (negative) impact on trade inflow (outflow) whereas credit quality has the opposite effect, suggesting a "free-from" rather than "flight-to" credit quality. Based on pre-crisis data, that study hence rejects the flight-to-quality (*i.e.* credit quality) hypothesis.

For the current crisis, however, based on our estimates of the risk premiums, the empirical evidence suggests that neither the flight-to-liquidity nor the flight-to-quality hypothesis can be rejected. We expect a significantly larger increase in the transaction costs of bonds that are deserted by investors than on save haven bonds. To test this hypothesis, we divide our sample of twelve issuers twice in two sub samples. In the first comparison, the average liquidity risk of the six issuers with the lowest liquidity risk is compared with the six issuers with the highest liquidity risk. In the second comparison, the average liquidity risk of the six issuers with the lowest credit risk is compared with the liquidity risk of the six issuers with the highest credit risk. The two comparisons are different because half of the issuers of the sample with lowest liquidity risk issuers are different from the sample with lowest credit risk issuers (see the notes of Table 5).

Table 5: Average change in the liquidity risk premium during the recent financial crisis, by risk category (in b.p.)

	5-year bonds			*10-year bonds*		
	Feb. 2006 – Jun. 2007	Jul. 2007 – Feb. 2010	Change (col. 3 – col. 2)	Feb. 2006 – Jun. 2007	Jul. 2007 – Feb. 2010	Change (col. 5 – col. 4)
Lowest liquidity risk	0.60 (0.03)	2.86 (0.63)	2.25	0.47 (0.02)	2.10 (0.42)	1.63
Highest liquidity risk	1.35 (0.46)	4.44 (0.86)	3.08	0.88 (0.25)	4.04 (1.10)	3.16
Lowest Credit risk	0.98 (0.40)	3.03 (0.90)	2.05	0.56 (0.07)	2.35 (0.69)	1.80
Highest Credit risk	0.98 (0.34)	4.26 (0.67)	3.28	0.79 (0.27)	3.79 (1.06)	3.00

Notes: Based on the LFA estimates of the factor loadings in shown Table 2. The standard error is within brackets. In the case of 5-year bonds, sovereign bonds of Austria, Finland, Germany, Italy, the Netherlands and Spain have the lowest liquidity risk whereas sovereign bonds of Belgium, Finland, France, Germany, Ireland and the Netherlands have the lowest credit risk over the period between Feb. 2010 and Jun. 2007. In the case of 10-year bonds, sovereign bonds of Belgium, Finland, France, Germany, Greece, and Italy bonds have the lowest liquidity risk whereas sovereign bonds of Finland, France, Germany, Ireland, the Netherlands and Spain have the lowest credit risk over the period between Feb. 2010 and Jun. 2007.

For both comparisons we find that liquidity risk increases faster for the group with the highest risk, were it liquidity risk or credit risk, suggesting that investors chase both liquidity and credit quality. The LRP of initially low liquidity risk issuers increased on average from 0.60 b.p. (0.47 b.p.) to 2.86 b.p. (2.10 b.p.) on 5-year (10-year) bonds (see Table 5). The LRP of initially low credit risk issuers increased on average from 0.98 b.p. (0.56 b.p.) to 3.03 b.p. (2.35 b.p.) on 5-year (10-year) bonds.

The LRP thus increased by about 2 b.p. for low liquidity and low credit risk issuers. For high liquidity and high credit risk issuers this increase was slightly above 3 b.p.. Hence, both high liquidity risk bonds and high credit risk bonds have become less attractive in comparison to low liquidity and low credit risk bonds respectively.

Let's next compare the order of the countries when sorted on the credit risk premium with the order of the countries when sorted on the liquidity risk premium during the crisis (see Figures 7a-7b and Figures 8a-8b respectively). While the cross-sectional relationship between the liquidity risk premium and credit risk premium is not one-to-one, countries with lower liquidity risk tend to have lower credit risk. For example, Dutch, French, and German bonds have both the lowest liquidity and credit risk. On the other side, both liquidity and credit risk are relatively high on Irish and Greek bonds. EIB bonds are rather exceptional in the sense that they have the highest liquidity risk but, at the same time, are among the group of lowest credit risk issuers.

The interaction between liquidity and credit risk was the topic of a recent study by Favero et al. (2010). The main idea brought forward by this work is that the demand for liquidity responds both to the magnitude of trading costs and to the availability of outside investment opportunities. It is assumed that investors are less likely to sell securities when outside investment opportunities are less attractive, a situation that is assumed to coincide with increased aggregate risk. Therefore, although high liquidity is positively valued by investors, they value it less when risk is high. Some empirical evidence in support of this new hypothesis is found for the pre-crisis years 2002 and 2003.

Our crisis sample results contrast with these predictions as, in addition to a positive cross-sectional relationship between liquidity risk and credit risk, we also find a strong positive relationship between the two risk components over time. The correlation coefficient is higher than 0.5 for most bonds (see Table 6). Liquidity is thus more valued when risk is high. During the recent crisis, higher aggregate risk reduced the value of new investment opportunities, which in turn led to lower credit demand and a lower CRF rate. As a result, the correlation between, on the one hand, the CRF rate, and, on the other hand, the liquidity risk or credit risk premium, was significantly negative. A full analysis of the drivers of liquidity risk premiums is beyond the scope of this paper.

Table 6: Correlations between the credit risk premium (CRP), the liquidity risk premium (LRP) and the common risk free (CRF) rate

	5-year bonds			*10-year bonds*		
	CRP and CRF	LRP and CRF	CRP and LRP	CRP and CRF	LRP and CRF	CRP and LRP
Austria	-0.84	-0.83	0.81	-0.81	-0.76	0.77
Belgium	-0.81	-0.82	0.83	-0.78	-0.80	0.89
Finland	-0.69	-0.84	0.43	-0.82	-0.78	0.58
France	-0.86	-0.81	0.85	-0.82	-0.74	0.87
Germany	-0.91	-0.73	0.71	-0.85	-0.65	0.68
Greece	-0.85	-0.81	0.78	-0.85	-0.63	0.74
Ireland	-0.88	-0.89	0.88	-0.83	-0.80	0.90
Italy	-0.83	-0.84	0.73	-0.87	-0.88	0.89
Netherlands	-0.86	-0.85	0.89	-0.81	-0.75	0.88
Portugal	-0.85	-0.76	0.67	-0.81	-0.72	0.76
Spain	-0.85	-0.85	0.79	-0.82	-0.79	0.90
EIB	-0.78	-0.36	0.08	-0.89	-0.35	0.46
Average	-0.83	-0.78	0.71	-0.83	-0.72	0.78

Notes: Based on the LFA-estimates of the factor loadings shown in Table 2. Sample period: Feb. 2006 – Feb. 2010.

4.4.3 Comparing the bond credit risk premium with the CDS credit risk premium

Finally, we analyse whether or not the price of credit risk is the same in the bond and the corresponding derivative market. In efficient markets, the exploitation of arbitrage opportunities would lead to a convergence of prices. One may therefore expect that the bond credit risk premium is on average equal to the CDS implied credit risk premium. The latter premium is computed by subtracting the CDS liquidity premium from the CDS rate. Figures 9a and 9b show that, during the crisis, the bond credit risk premium is substantially higher than the CDS implied credit risk premium for most of the bonds (no data is available on the transaction cost of the EIB CDSs). For example, the difference exceeds 25 b.p. for Finnish and Greek bonds between Jul. 2007 and Feb. 2010. Before the crisis, the 5-year bond CRP was broadly in line with the corresponding CDS CRP. The CRP on some of the 10-year bonds however was substantially above the corresponding CDS CRP. Over the full period of analysis, the price of sovereign credit risk is about the same (*i.e.* the difference is 5 b.p. or less) in both the bond and CDS market for only two countries, *i.e.* Germany and Spain.

Fig. 9a: Average 5-year bond CRP minus CDS CRP (in b.p.)

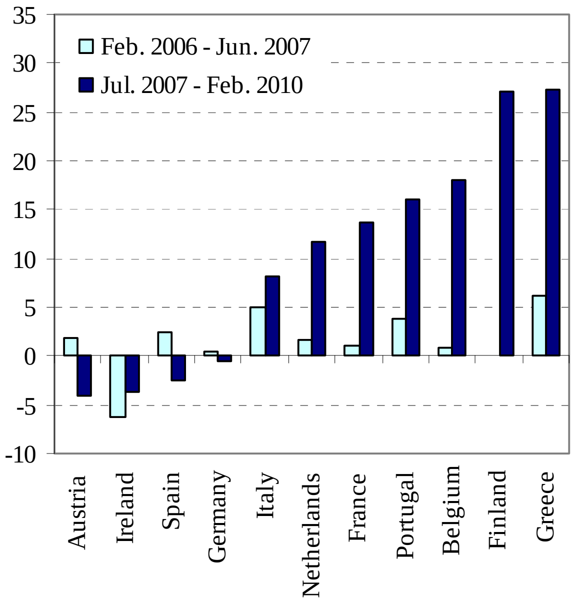
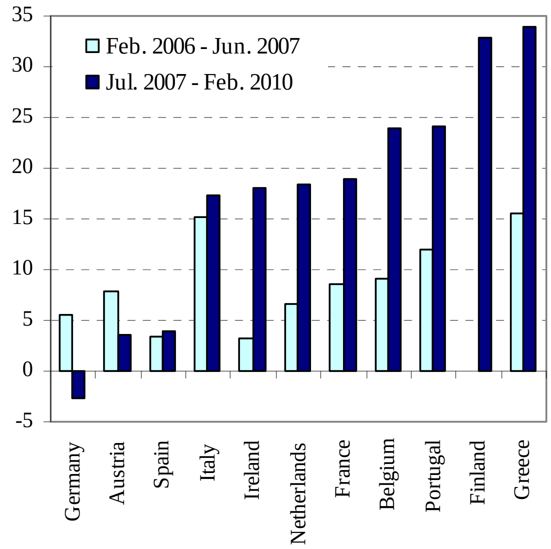


Fig. 9b: Average 10-year bond CRP minus CDS CRP (in b.p.)



Notes: Based on the LFA-estimates of the factor loadings shown in Table 2.

These results imply that 1) if the CDS implied credit risk premium is in line with the fundamentals, then many euro area sovereign bonds were under-priced during the crisis, and, thus, borrowing costs were higher than warranted on the basis of true creditworthiness; 2) if the bond price is in line with the fundamentals, then many CDS were under-priced, and, thus, the cost of insurance against default was lower than warranted on the basis of true creditworthiness; or a combination of both 1) and 2). It is also possible that discrepancies between quotes and actual transaction prices were more important in one market than the other.

5 Conclusion

We introduce Longitudinal Factor Analysis to extract the common risk free rate from a sample of sovereign bonds of countries in a monetary union. Cross-variable restrictions are derived that are ignored by classical factor analysis. Since LFA exploits the typically very large longitudinal dimension of bond data, it performs better than traditional methods. A Monte Carlo experiment shows that substantial efficiency gains can be made in the estimation of idiosyncratic risk and factor loadings. The factor loading concomitant the benchmark security is required to determine the volatility of the CRF rate. Its level is determined by an auxiliary regression.

The bond yield decomposition procedure proposed in this paper sheds new light on some key issues in the euro area sovereign bond markets such as benchmark status, flight-to-liquidity and flight-to-quality hypotheses, and price differences between the bond and derivative markets. First, our empirical findings suggest that since the 2007 financial crisis the German Bund has the lowest idiosyncratic risk, and hence benchmark status. The fact that the German bund did not have the lowest idiosyncratic risk before the crisis confirms the findings of Dunne et al. (2007). Second, in contrast with Beber et al. (2009), who interpret a negative relationship between credit quality and trade inflow as a sign that investors are less concerned by credit quality than liquidity, our results suggest that investors chase both credit quality and liquidity in episodes of market stress. The liquidity risk premium increased more for both *ex ante* high liquidity risk and high credit risk issuers than for *ex ante* low liquidity risk and low credit risk issuers. Third, in contrast with the prediction of the Favero et al. (2010) model, liquidity is more valued when aggregate risk is high as, in addition to a positive cross-sectional relationship between liquidity risk and credit risk, we also find a strong positive relationship between the two risk components over time. Finally, given the cost of insurance against default, many euro area sovereign bonds seem under-priced in the recent financial turmoil.

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Appendix I: The likelihood function

The integral in Equation (13) can be developed as:

$$\int_{Z_{t=-\infty}}^{\infty} \prod_{l\in\{i,j,k\}} e^{-\frac{1}{2}\frac{(x_{lt}-a_l-b_l Z_t)^2}{s_l^2}} dZ_t = \int_{Z_{t=-\infty}}^{\infty} e^{-\frac{1}{2}\left(\frac{b_i^2}{s_i^2}+\frac{b_j^2}{s_j^2}+\frac{b_k^2}{s_k^2}\right)\left(Z_t-\frac{\frac{b_i}{s_i^2}(x_{it}-a_i)+\frac{b_j}{s_j^2}(x_{jt}-a_j)+\frac{b_k}{s_k^2}(x_{kt}-a_k)}{\frac{b_i^2}{s_i^2}+\frac{b_j^2}{s_j^2}+\frac{b_k^2}{s_k^2}}\right)^2} dZ_t \times$$

$$e^{-\frac{1}{2}\left(\frac{(x_{it}-a_i)^2}{s_i^2}+\frac{(x_{jt}-a_j)^2}{s_j^2}+\frac{(x_{kt}-a_k)^2}{s_k^2}\right)+\frac{1}{2}\frac{\left(\frac{b_i}{s_i^2}(x_{it}-a_i)+\frac{b_j}{s_j^2}(x_{jt}-a_j)+\frac{b_k}{s_k^2}(x_{kt}-a_k)\right)^2}{\frac{b_i^2}{s_i^2}+\frac{b_j^2}{s_j^2}+\frac{b_k^2}{s_k^2}}} \qquad (18)$$

Equation (18) shows that Z_t is perceived as having a normal distribution with mean:

$$E[Z_t] = \frac{\frac{b_i^2}{s_i^2}\frac{x_{it}-a_i}{b_i}+\frac{b_j^2}{s_j^2}\frac{x_{jt}-a_j}{b_j}+\frac{b_k^2}{s_k^2}\frac{x_{kt}-a_k}{b_k}}{\frac{b_i^2}{s_i^2}+\frac{b_j^2}{s_j^2}+\frac{b_k^2}{s_k^2}}, \qquad (19)$$

and variance:

$$V(Z_t) = (b_i^2/s_i^2 + b_j^2/s_j^2 + b_k^2/s_k^2)^{-1}. \qquad (20)$$

The value of the integral is thus $(2\pi)^{1/2}$ divided by the standard deviation of Z_t. The total likelihood in Equation (18) is now given by:

$$\prod_{t=1}^{T} \frac{1}{2\pi s_i s_j s_k} \frac{1}{\sqrt{\frac{b_i^2}{s_i^2}+\frac{b_j^2}{s_j^2}+\frac{b_k^2}{s_k^2}}} e^{-\frac{1}{2}\left(\frac{(x_{it}-a_i)^2}{s_i^2}+\frac{(x_{jt}-a_j)^2}{s_j^2}+\frac{(x_{kt}-a_k)^2}{s_k^2}\right)+\frac{1}{2}\frac{\left(\frac{b_i}{s_i^2}(x_{it}-a_i)+\frac{b_j}{s_j^2}(x_{jt}-a_j)+\frac{b_k}{s_k^2}(x_{kt}-a_k)\right)^2}{\frac{b_i^2}{s_i^2}+\frac{b_j^2}{s_j^2}+\frac{b_k^2}{s_k^2}}} \qquad (21)$$

If we would allow the implied variability of the exogenous process to vary, we should correct the likelihood for this (this is similar to a uniformly distributed random variable on a certain interval: the higher its variability, the longer the interval and the lower the density). We avoid this correction by using a standardization that keeps the variability of the exogenous process constant.

To obtain the maximizing values for a_i we set the respective derivative of the likelihood function equal to 0 which yields:

$$\sum_{t=1}^{T} \frac{x_{it} - a_i}{s_i^2} - \frac{\left(\frac{b_i}{s_i^2}\left(x_{it} - a_i\right) + \frac{b_j}{s_j^2}\left(x_{jt} - a_j\right) + \frac{b_k}{s_k^2}\left(x_{kt} - a_k\right) \right)}{\frac{b_i^2}{s_i^2} + \frac{b_j^2}{s_j^2} + \frac{b_k^2}{s_k^2}} \frac{b_i}{s_i^2} = 0 \quad . \tag{22}$$

This condition can be written as:

$$\frac{\overline{E}_t\left[x_{it}\right] - a_i}{b_i} = \frac{\frac{b_j^2}{s_j^2} \frac{\overline{E}_t\left[x_{jt}\right] - a_j}{b_j} + \frac{b_k^2}{s_k^2} \frac{\overline{E}_t\left[x_{kt}\right] - a_k}{b_k}}{\frac{b_j^2}{s_j^2} + \frac{b_k^2}{s_k^2}} \quad . \tag{23}$$

In other words, $(\overline{E}_t\left[x_{it}\right] - a_i) / b_i$ is a weighted average of the likewise terms for bonds j and k. Taking the derivatives to a_j and to a_k similarly gives that each term is a weighted average of the two other terms. But then all terms have to be identical as claimed in Equation (15).

Appendix II: Monte Carlo results

Table 7: Average factor loadings and average variances

	True value	PC	EM	LFA
	Factor loadings b_i, $i = 1, ..., 12$			
Bond 1	1.000	0.986	0.996	0.999
Bond 2	1.100	1.087	1.098	1.099
Bond 3	1.200	1.188	1.199	1.198
Bond 4	1.300	1.291	1.300	1.298
Bond 5	1.400	1.395	1.401	1.398
Bond 6	1.500	1.501	1.502	1.498
Bond 7	1.600	1.609	1.604	1.598
Bond 8	1.700	1.719	1.705	1.698
Bond 9	1.800	1.831	1.806	1.798
Bond 10	1.900	1.947	1.908	1.898
Bond 11	2.000	2.066	2.011	1.998
Bond 12	2.100	2.187	2.113	2.098
	Variance of the errors, s_i^2, $i = 1, ...,12$			
Bond 1	0.040	0.067	0.050	0.040
Bond 2	0.090	0.119	0.097	0.090
Bond 3	0.160	0.188	0.164	0.160
Bond 4	0.250	0.273	0.252	0.250
Bond 5	0.360	0.373	0.359	0.360
Bond 6	0.490	0.486	0.485	0.489
Bond 7	0.640	0.610	0.632	0.639
Bond 8	0.810	0.743	0.796	0.809
Bond 9	1.000	0.883	0.980	0.999
Bond 10	1.210	1.027	1.182	1.208
Bond 11	1.440	1.172	1.402	1.439
Bond 12	1.690	1.314	1.639	1.688

Notes: Averages are computed over ten thousand replications. For each replication twelve bond yield series with one thousand observations each are generated as follows: $x_{it} = b_i Z_t + e_{it}$, $b_i = 1 + (i-1)/10$, $s_i^2 = (0.2 + (i-1)/10)^2$, $i \in \{1,...,12\}$. Z_t is drawn from a standard normal distribution. e_{it} is drawn from a normal distribution with mean zero and variance equal to s_i^2. The Principal Components (PC) results in the middle column are obtained with Theil's (1971) method. Classical factor analysis results are obtained with the EM algorithm of Rubin and Thayer (1982). The factor analysis results in the last column are obtained with the LFA method developed in Section 3.

Appendix III: Construction of constant maturity bond yields

Constant 5-year and 10-year maturity bond spot yields are derived from linear inter- and extrapolation of secondary market plain vanilla bond quotes on Bloomberg. We first interpolate the *z*-spreads reported on Bloomberg, and then add the interpolated *z*-spread to the swap spot rates in order to compute the bond spot rate with exact maturity m. For example, the *z*-spread for maturity m concomitant country i in period t is estimated by:

$$z_{it,m} = w \times z_{it,m_1} + (1-w) \times z_{it,m_2}, \qquad (24)$$

where w is the weight on the *z*-spread z_{it,m_1} concomitant country i's bond with maturity m_1, and $(1-w)$ is the weight on the *z*-spread z_{it,m_2} concomitant country i's bond with maturity m_2; $w = (m_2 - m)/(m_2 - m_1)$; $m \in \{5, 10\}$. Since we restrict our analysis to two benchmark maturities only, full yield curve modelling with *a priori* chosen functional forms as in Nelson and Siegel (1987) is not necessarily best. Measurement errors (in the *z*-spreads z_{it,m_1} and z_{it,m_2}) may be less distorting in the latter approach, but our approach is less restrictive concerning the curvature of the yield curve around the benchmark maturities, and thus potentially better in estimating the benchmark maturity spot rates.

Preferably, the on-the-run bond is used when inter- and extrapolating. For a bond to be considered an on-the-run bond in period t its remaining time to maturity must be closer to m years than any other bond and its original maturity should be in the reference interval $(m - \tau_m, m + \tau_m)$. The values $\tau_5 = 1$ and $\tau_{10} = 2$ are used, *i.e.* for a bond to be considered an on-the-run 5-year bond it must expire within four to six years (*i.e.* not before $t^* + 4$ and not after $t^* + 6$) after the issuance date (t^*).

In addition, the following selection criteria are applied:

1) If the on-the-run bond exists, the closest bond to the reference maturity at the opposite side of the on-the-run bond is chosen, and we interpolate. If the latter is not available, then the closest bond at the side of the on-the-run bond is chosen, and we extrapolate.

2) If in period t there is no on-the-run bond then the two bonds closest to the reference maturity at opposite sides are chosen, and we interpolate. If one side is not available, the two closest bonds at one side are chosen, and we extrapolate.

3) Only long-term fixed rate bonds are included. The original maturity of the selected bonds exceeds four years.

4) French, German and Italian bonds are only selected when they have a minimum size of € 1 billion. For all other countries and the European Investment Bank the minimum size is € 500 million.

5) Yields of bonds are excluded during their last year (*i.e.* short-term instruments are excluded). Also, days during the first month are excluded if the bid-ask spread on the yield to maturity is larger than 15 b.p..

6) Only plain vanilla fixed coupon bonds are selected. All securities with option-like features, including callable bonds, are excluded.

7) Outliers are removed.[9]

[9] Four outlier bonds (*i.e.* two Irish bonds, one issued on 4/11/1986 and one issued on 18/8/1994, and two EIB bonds, one issued on 20/2/1997 and one issued on 16/2/1998), two outlier yields on the included EIB bond issued on 6/4/2006, and all bond yields on 17/4/2006, 19/6/2006 and 21/03/2008 have been removed.

Appendix IV: Descriptive statistics

Table 8: Descriptive statistics of the bond data (Feb. 2006 – Feb. 2010)

	5-year bonds					
	Number of bonds used in the estimation	Original maturity (in years)		Size (in € bn)		
		Min	Max	Min	Avg	Max
Austria	7	5.8	15.0	1.3	7.6	12.2
Belgium	9	5.9	20.0	7.5	11.0	15.8
Finland	6	5.3	11.2	5.0	5.7	6.5
France	19	5.0	25.8	3.4	15.3	20.0
Germany	20	5.0	10.2	7.0	19.7	27.0
Greece	13	5.2	15.0	2.5	7.4	12.5
Ireland	5	5.0	16.9	5.8	6.5	8.2
Italy	15	4.9	10.6	4.0	18.6	28.3
Netherlands	7	5.5	10.5	8.0	12.1	15.5
Portugal	8	5.4	15.3	4.3	5.8	7.1
Spain	13	5.3	15.6	1.3	11.2	15.8
EIB	8	5.0	10.4	0.5	5.0	9.8
Number of observations per issuer			1058			
	10-year bonds					
	Number of bonds used in the estimation	Original maturity (in years)		Size (in € bn)		
		Min	Max	Min	Avg	Max
Austria	9	10.0	15.5	1.3	8.1	12.2
Belgium	8	10.2	19.7	4.0	8.6	12.2
Finland	6	11.0	15.7	3.0	5.0	6.5
France	13	10.0	30.7	5.4	16.6	22.0
Germany	13	10.1	30.0	3.8	17.6	24.0
Greece	9	10.2	20.5	4.6	9.0	15.5
Ireland	7	10.3	16.9	5.0	6.5	8.2
Italy	15	10.1	15.9	5.0	20.0	25.2
Netherlands	10	10.0	30.0	6.4	10.5	15.5
Portugal	8	10.2	16.1	3.0	5.6	6.9
Spain	11	10.3	31.0	3.0	11.0	15.0
EIB	6	10.0	15.8	3.3	5.0	7.0
Number of observations per issuer			1058			

Source: Own calculations based on Bloomberg.

Table 9: Descriptive statistics of bond spot yields (in %, Feb. 2006 – Feb. 2010)

	Min	Avg	Median	Max	St dev
			5-year bonds		
Austria	2.41	3.67	3.72	4.87	0.55
Belgium	2.46	3.69	3.74	4.96	0.58
Finland	2.28	3.60	3.71	4.87	0.62
France	2.32	3.57	3.71	4.88	0.65
Germany	2.14	3.44	3.67	4.74	0.73
Greece	3.31	4.28	4.18	6.97	0.60
Ireland	3.10	3.97	3.93	5.20	0.41
Italy	2.77	3.86	3.90	5.14	0.54
Netherlands	2.31	3.58	3.71	4.87	0.62
Portugal	2.78	3.82	3.85	4.98	0.48
Spain	2.68	3.70	3.74	4.90	0.54
EIB	2.52	3.76	3.82	5.04	0.61
T			1058		
			10-year bonds		
	Min	Avg	Median	Max	St dev
Austria	3.46	4.14	4.14	4.88	0.30
Belgium	3.48	4.17	4.13	4.99	0.32
Finland	3.42	4.06	4.03	4.86	0.32
France	3.47	4.04	4.02	4.85	0.34
Germany	2.97	3.86	3.93	4.69	0.41
Greece	3.71	4.78	4.65	7.31	0.63
Ireland	3.45	4.53	4.47	6.17	0.57
Italy	3.71	4.42	4.42	5.32	0.30
Netherlands	3.45	4.06	4.04	4.85	0.32
Portugal	3.57	4.34	4.35	5.08	0.32
Spain	3.47	4.19	4.15	4.94	0.29
EIB	3.54	4.21	4.19	5.01	0.33
T			1058		

Source: Own calculations based on Bloomberg.

Table 10: Descriptive statistics of CDS spreads (in b.p., Feb. 2006 – Feb. 2010)

	Min	Avg	Median	Max	St dev	*T*
5-year CDS						
Austria	0.5	39.2	7.7	273.0	55.2	1057
Belgium	1.0	28.0	17.2	157.8	33.5	1057
Finland	6.5	32.2	27.9	93.9	20.0	470
France	0.5	18.2	8.8	97.7	21.4	1057
Germany	0.6	15.5	5.8	91.9	18.9	1057
Greece	4.4	80.5	37.4	428.3	95.3	1057
Ireland	1.5	72.1	20.8	395.8	91.4	1057
Italy	5.3	50.9	28.5	200.6	52.1	1057
Netherlands	1.0	21.8	8.2	131.0	29.4	1053
Portugal	3.4	41.2	27.6	244.4	43.3	1057
Spain	1.8	42.3	26.8	173.4	44.7	1054
EIB	2.5	21.4	7.8	68.0	18.3	592
10-year CDS						
	Min	Avg	Median	Max	St dev	*T*
Austria	0.8	41.2	12.0	260.1	53.6	1057
Belgium	2.4	31.4	23.3	152.9	33.1	1057
Finland	11.0	35.6	32.8	94.2	19.1	470
France	1.4	20.9	13.0	96.6	21.6	1048
Germany	0.7	17.7	9.0	90.7	18.9	1048
Greece	10.8	87.0	47.2	379.9	87.0	1057
Ireland	2.3	73.5	26.8	365.0	87.2	1057
Italy	11.4	58.8	38.5	205.3	48.5	1057
Netherlands	1.8	24.9	12.3	126.3	29.6	1018
Portugal	7.2	46.7	36.8	227.1	40.7	1057
Spain	4.4	46.8	35.3	169.0	43.7	1057
EIB	3.5	24.3	14.0	78.0	19.8	481

Source: Own calculations based on sovereign and EIB CDS spreads from Credit Market Analysis Limited and Markit respectively.

Table 11: Average bond and CDS transaction costs (in b.p.)

	5-year bond and CDS					
	Feb. 2006 – Jun. 2007			Jul. 2007 – Feb. 2010		
	Bond	*CDS*	*CDS - Bond*	*Bond*	*CDS*	*CDS – Bond*
Austria	0.4	1.5	1.1	3.4	4.7	1.3
Belgium	0.4	1.4	1.0	2.0	4.6	2.6
Finland	0.4	n.a.	n.a.	2.2	5.3	3.1
France	0.4	1.3	0.9	1.4	3.5	2.1
Germany	0.3	1.3	1.0	0.6	3.1	2.5
Greece	0.4	1.5	1.1	2.7	7.0	4.3
Ireland	1.8	2.3	0.5	4.5	6.9	2.5
Italy	0.4	1.2	0.8	1.7	4.2	2.5
Netherlands	0.4	1.6	1.2	1.3	4.8	3.4
Portugal	0.5	1.5	1.0	2.5	4.4	1.8
Spain	0.4	2.2	1.8	2.1	4.2	2.1
EIB	1.7	n.a.	n.a.	4.5	n.a.	n.a.
Average	0.6	1.6	1.0	2.4	4.8	2.6
T	357			678		
	10-year bond and CDS					
	Feb. 2006 – Jun. 2007			Jul. 2007 – Feb. 2010		
	Bond	*CDS*	*CDS - Bond*	*Bond*	*CDS*	*CDS - Bond*
Austria	0.4	1.9	1.5	2.0	4.9	2.9
Belgium	0.3	2.9	2.6	1.2	4.6	3.4
Finland	0.3	n.a.	n.a.	1.5	5.4	3.9
France	0.2	2.9	2.6	1.0	3.5	2.5
Germany	0.2	2.3	2.0	0.4	3.0	2.6
Greece	0.3	3.9	3.6	2.1	7.3	5.3
Ireland	0.5	2.3	1.8	3.0	7.2	4.2
Italy	0.3	3.9	3.6	1.1	4.4	3.3
Netherlands	0.3	1.3	1.0	0.9	4.8	3.9
Portugal	0.4	3.2	2.9	1.9	4.4	2.6
Spain	0.3	1.2	0.9	1.3	4.3	3.0
EIB	1.2	n.a.	n.a.	4.8	n.a.	n.a.
Average	0.4	2.6	2.2	1.8	4.9	3.4
T	357			678		

Source: Own calculations based on sovereign bond and CDS quotes from Bloomberg (price provider = bond trader composite) and Credit Market Analysis Limited respectively.
Note: The transaction cost is measured by the difference in the bid and the ask rate.