Appendix to Chapter 6: From trade liberalization to productivity growth

Philippe Aghion *

In the previous sections of the paper we discussed the impact of productivity growth on firms' ability to export and more generally be "open". Here we discuss the reversed causality from trade openness to productivity growth.

1 The one-country benchmark

1.1 Production and profits

Consider first the benchmark case of a single country in which a unique final good, which also serves as numéraire, is produced competitively using a continuum of intermediate inputs according to:

$$Y_t = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di, \; 0 < \alpha < 1 \tag{1}$$

where L is the domestic labor force, assumed to be constant, A_{it} is the quality of intermediate good i at time t, and x_{it} is the flow quantity of intermediate good i being produced and used at time t.

In each intermediate sector there is a monopolist producer who uses the final good as the sole input, with one unit of final good needed to produce each unit of intermediate good. The monopolist's cost of production is therefore equal to the quantity produced x_{it}. The price p_{it} at which she can sell this quantity of intermediate good to the competitive final sector is the marginal product of intermediate good i in the final good production function (1)

As we show in Aghion and Howitt (2009), the monopolist will choose the level of output that maximizes profits, namely

$$x_{it} = A_{it} L \alpha^{2/(1-\alpha)} \tag{2}$$

resulting in the profit level

$$\pi_{it} = \pi A_{it} L \tag{3}$$

where $\pi \equiv (1-\alpha)\, \alpha^{\frac{1+\alpha}{1-\alpha}}$.

The equilibrium level of final output in the economy can be found by substituting the x_{it}'s into (1), which yields

$$Y_t = \varphi A_t L \tag{4}$$

*This document contains the appendix to chapter 6 of the European Investment Bank's publication Investment and Investment Finance in Europe 2015.

where A_t is the average productivity parameter across all sectors

$$A_t = \int_0^1 A_{it} di$$

and $\varphi = \alpha^{\frac{2\alpha}{1-\alpha}}$.[1]

We are interested in the equilibrium level of national income, N_t, which is not quite the same as final sector output Y_t, because some of the final goods are used up in producing the intermediate products which in turn are used in producing the final output. There are only two forms of income - wage income and profit income. Each of the L workers is employed in the final goods sector, where he is paid his marginal product. So according to the production function (1), total wage income is the fraction $1-\alpha$ of final output:

$$W_t = L \times \partial Y_t \partial / L = (1-\alpha) Y_t$$

Profits are earned by the local monopolists who sell intermediate products to the final sector. (There are no profits in the final good sector because that sector is perfectly competitive and operates under constant returns to scale). Since each monopolist charges a price equal to[2] $1/\alpha$ and has a cost per unit equal to 1, therefore the profit margin $p_{it}-1$ on each unit sold can be written as $(1-\alpha) p_{it}$, which means that total profits equal

$$\Pi_t = \int_0^1 (p_{it}-1) x_{it} dt = (1-\alpha) \int_0^1 p_{it} x_{it} dt$$

And since the price p_{it} is the marginal product of the ith intermediate product, therefore (1) implies:

$$\int_0^1 p_{it} x_{it} dt = \int_0^1 (\partial Y_t / \partial x_{it}) x_{it} dt = \alpha Y_t$$

So from the last two lines we have

$$\Pi_t = (1-\alpha) \alpha Y_t$$

Gathering results we see that national income is

$$N_t = W_t + \Pi_t = \left(1-\alpha^2\right) Y_t. \tag{5}$$

[1] To derive this expression for Y_t, substitute the x_{it}'s into (1) to get

$$\begin{aligned} Y_t &= L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} \left(A_{it} L \alpha^{2/(1-\alpha)}\right)^\alpha di \\ &= \left(\alpha^{2/(1-\alpha)}\right)^\alpha L \int_0^1 A_{it}^{1-\alpha} A_{it}^\alpha di \\ &= \zeta A_t L \end{aligned}$$

[2] That is, since the final-good market in which the intermediate product is sold is perfectly competitive, therefore its price equals its marginal product:

$$p_{it} = \partial Y_t / \partial x_{it} = \alpha L^{1-\alpha} A_t^{1-\alpha} x_{it}^{\alpha-1}$$

Using (2) to substitute for x_{it} in this equation yields $p_{it} = 1/\alpha$.

From (4) and (5) we see that national income is strictly proportional to average productivity and to population:

$$N_t = \left(1-\alpha^2\right)\varphi A_t L \tag{6}$$

It follows from this last result that the growth rate of national income is also the growth rate of productivity

$$\dot{N}_t/N_t = \dot{A}_t/A_t = g_t.$$

1.2 Innovation

Productivity growth comes from innovations. In each sector, at each date there is a unique entrepreneur with the possibility of innovating in that sector. She is the incumbent monopolist, and an innovation would enable her to produce with a productivity (quality) parameter $A_{it} = \gamma A_{i,t-1}$ that is superior to that of the previous monopolist by the factor $\gamma > 1$. Otherwise her productivity parameter stays the same: $A_{it} = A_{i,t-1}$. In order to innovate with any given probability μ she must spend the amount

$$c_{it}(\mu) = (1-\tau)\cdot\phi\left(\mu\right)\cdot A_{i,t-1}$$

of final good in research, where $\tau > 0$ is a subsidy parameter that represents the extent to which national policies encourage innovation, and ϕ is a cost function satisfying

$$\begin{aligned} \phi\left(0\right) &= 0;\ \text{and} \\ \phi'\left(\mu\right) &> 0,\ \phi''\left(\mu\right) > 0 \text{ for all } \mu > 0 \end{aligned}$$

Thus the local entrepreneur's expected profit net of research cost is

$$\begin{aligned} V_{it} &= E\pi_{it} - c_{it}(\mu) \\ &= \mu\pi L\gamma A_{i,t-1} + (1-\mu)\,\pi L A_{i,t-1} - (1-\tau)\,\phi\left(\mu\right)A_{i,t-1} \end{aligned}$$

She will choose the value of μ that maximizes these expected profits.

Each local entrepreneur will choose a frequency of innovations μ^* that maximizes V_{it}. The first-order condition for an interior maximum is $\partial V_{it}/\partial\mu = 0$, which can be expressed as the research arbitrage equation:

$$\phi'\left(\mu\right) = \pi L\left(\gamma-1\right)/\left(1-\tau\right). \tag{7}$$

If the research environment is are favorable enough (i.e. if τ is large enough), or the population large enough, so that:

$$\phi'\left(0\right) > \pi L\left(\gamma-1\right)/\left(1-\tau\right)$$

then the unique solution μ to the research arbitrage equation (7) is positive, so in each sector the probability of an innovation is that solution ($\widehat{\mu} = \mu$), which is an increasing function of the size of population L and of the policies favoring innovation τ. Otherwise there is no positive solution to the research arbitrage equation so the local entrepreneur chooses never to innovate ($\widehat{\mu} = 0$).

Since each A_{it} grows at the rate $\gamma - 1$ with probability $\widehat{\mu}$, and at the rate 0 with probability $1 - \widehat{\mu}$, therefore the expected growth rate of the economy is

$$g = \widehat{\mu}(\gamma - 1)$$

So we see that countries with a larger population and more favorable innovation conditions will be more likely to grow, and if they grow will grow faster, than countries with a smaller population and less favorable innovation conditions.

2 Opening up to trade

Now, let us open trade in goods (both intermediate and final) between the domestic country and the rest of the world. To keep the analysis simple, suppose that there are just two countries, called "home" and "foreign", which differ in terms of the size of population and the policies favoring innovation. Suppose that the range of intermediate products in each country is identical, that they produce exactly the same final product, and that there are no transportation costs. Within each intermediate sector the world market can then be monopolized by the producer with the lowest cost. We use asterisks to denote foreign-country variables.

2.1 Production and profits

Consider the following conceptual experiment. To begin with, each country does no trade, and hence behaves just like the closed economy described in the previous section. Then at time t we allow them to trade costlessly with each other. The immediate effect of this opening up is to allow each country to take advantage of more efficient productive efficiency. In the home country, final good production will equal

$$Y_t = \int_0^1 Y_{it} di = L^{1-\alpha} \int_0^1 \widehat{A}_{it}^{1-\alpha} x_{it}^{\alpha} di, \ 0 < \alpha < 1 \tag{8}$$

where $\widehat{A}_{it}$ is the higher of the two initial productivity parameters:

$$\widehat{A}_{it} = \max\{A_{it}, A_{it}^*\}$$

Likewise in the foreign country final good production will equal

$$Y_t^* = \int_0^1 Y_{it}^* di = (L^*)^{1-\alpha} \int_0^1 \widehat{A}_{it}^{1-\alpha} (x_{it}^*)^{\alpha} di, \ 0 < \alpha < 1 \tag{9}$$

The profit of a monopolist will now be higher than it was under autarky, because she can now sell to both countries. Specifically, if she charges the price p_{it} then final good producers in each country will buy good i up to the point where its marginal product equals p_{it}:

$$x_{it} = \widehat{A}_{it} L (p_{it}/\alpha)^{\frac{1}{\alpha-1}} \text{ and } x_{it}^* = \widehat{A}_{it} L^* (p_{it}/\alpha)^{\frac{1}{\alpha-1}} \tag{10}$$

Adding these two equations are rearranging, we see that her price will depend on total sales $X_{it} = x_{it} + x_{it}^*$ according to:

$$p_{it} = \alpha (L + L^*)^{1-\alpha} \left(\widehat{A}_{it}\right)^{1-\alpha} X_{it}^{\alpha-1} \tag{11}$$

which is the same as the demand relationship in the closed economy[3] except that now we have global sales relative to global population instead of local sales relative to local population on the right hand side. Accordingly the monopolist's profit π_{it} will equal revenue $p_{it}X_{it}$ minus cost X_{it}:

$$\pi_{it} = p_{it}X_{it} - X_{it} = \alpha\left(\widehat{A}_{it}\right)^{1-\alpha}(L+L^*)^{1-\alpha}X_{it}^{\alpha} - X_{it}$$

As in the case of the closed economy, the monopolist will choose the level of output that maximizes π_{it}, namely

$$X_{it} = \widehat{A}_{it}\,(L+L^*)\,\alpha^{2/(1-\alpha)}$$

resulting in the same price

$$p_{it} = 1/\alpha$$

as before and the profit level

$$\pi_{it} = \pi\widehat{A}_{it}\,(L+L^*) \tag{12}$$

where once again $\pi \equiv (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}$.

Substituting the prices $p_{it} = 1/\alpha$ into the demand functions (10) yields

$$x_{it} = \widehat{A}_{it}L\alpha^{2/(1-\alpha)} \text{ and } x_{it}^* = \widehat{A}_{it}L^*\alpha^{2/(1-\alpha)}$$

and substituting these into the production functions (8) and (9) we see that final good production in the two countries will be proportional to their populations:

$$Y_t = \varphi\widehat{A}_tL \text{ and } Y_t^* = \varphi\widehat{A}_tL^* \tag{13}$$

where once again $\varphi = \alpha^{\frac{2\alpha}{1-\alpha}}$ and where $\widehat{A}_t$ is the cross-sectoral average of the $\widehat{A}_{it}$'s:

$$\widehat{A}_t = \int_0^1 \widehat{A}_{it}di$$

2.2 Step-by-step innovation

Consider the innovation process in a given sector i. In the country where the monopoly currently resides, the country is on the global technology frontier for sector i, and the local entrepreneur will aim at making a frontier innovation that raises the productivity parameter from $\widehat{A}_{it}$ to $\gamma\widehat{A}_{it}$. If so, that country will retain a global monopoly in intermediate product i. In the other country, the local entrepreneur will be trying to catch up with the frontier by implementing the current frontier technology. If she succeeds and the frontier entrepreneur fails to advance the frontier that period, then the lagging country will have caught up, both countries will be on the frontier, and we can suppose that each entrepreneur will monopolize the market for product i in her own country. But if the frontier entrepreneur does advance the frontier then the entrepreneur in the lagging country will still remain behind and will earn no profit income.

Over time, the lead in each sector will tend to pass from country to country, as long as the lagging sector is innovating. (Otherwise the lead will remain the

[3]See footnote 2 above.

country that starts with the lead when trade is opened up.) However there will be no immediate leapfrogging of one country by the other, because in order to retake the lead a country must first catch up. So in between lead changes there will be a period when the sector is level, or neck-and-neck, as shown in Aghion and Howitt (2009, Chapter 12) when we study the effect of product market competition on innovation. As in that chapter, the growth rate of productivity will be determined by the incentives to perform R&D in the different cases (when the country is the sole leader, when it is the laggard, and when the sector is level.) So we need to study each case in turn.

Three possibilities must be considered. Either a domestic sector leads over the corresponding sector in the foreign country (case A); or the domestic sector is at level (neck-and-neck) with its counterpart in the foreign country (case B); or the domestic sector lags behind its foreign counterpart (case C). More precisely.

A Case A is the case in which the lead in sector i resides in the home country, while the foreign country lags behind. In this case the expected profit of the entrepreneur in the home country, net of R&D costs, is

$$EU_A = \mu_A \gamma \left(L + L^*\right) \pi + \left(1 - \mu_A\right) \left(L + \left(1 - \mu_A^*\right) L^*\right) \pi - \left(1 - \tau\right) \phi\left(\mu_A\right)$$

while the expected profit of the foreign entrepreneur is

$$EU_A^* = \mu_A^* \left(1 - \mu_A\right) \pi L^* - \left(1 - \tau^*\right) \phi\left(\mu_A^*\right)$$

where everything is normalized by the pre-existing productivity level. That is, with probability μ_A the home entrepreneur will innovate, thus earning all the global profits in the market at productivity level γ times the pre-existing level; if she fails to innovate then she will still earn all domestic profits in the market, at the pre-existing profit level, and if the foreign entrepreneur fails to innovate (which occurs with probability $1 - \mu_A^*$) she will also earn all the foreign profits in the market. In any event she must incur the R&D cost $\left(1 - \tau\right) \phi\left(\mu_A\right)$. Likewise the foreign entrepreneur will earn all the profits in the foreign market if she innovates and her rival doesn't, which occurs with probability $\mu_A^* \left(1 - \mu_A\right)$.

B Case B is the case in which the sector is level. In this case the expected profits of the respective entrepreneurs net of R&D costs are

$$\begin{aligned} EU_B &= \left(\mu_B \left(L + \left(1 - \mu_B^*\right) L^*\right) \gamma + \left(1 - \mu_B\right)\left(1 - \mu_B^*\right) L\right) \pi - \left(1 - \tau\right) \phi\left(\mu_B\right) \text{ and} \\ EU_B^* &= \left(\mu_B^* \left(L^* + \left(1 - \mu_B\right) L\right) \gamma + \left(1 - \mu_B^*\right)\left(1 - \mu_B\right) L^*\right) \pi - \left(1 - \tau^*\right) \phi\left(\mu_B^*\right) \end{aligned}$$

That is, for example, the home entrepreneur innovates with probability μ_B, which earns her all the home profits for sure and all the foreign profits if her rival fails to innovate, whereas if both fail to innovate then she retains all the domestic profits.

C Case C is the case in which the foreign country starts with the lead. By analogy with case A the expected profits minus R&D costs are respectively:

$$\begin{aligned} EU_C &= \mu_C \left(1 - \mu_C^*\right) \pi L - \left(1 - \tau\right) \phi\left(\mu_C\right) \text{ and} \\ EU_C^* &= \mu_C^* \gamma \left(L + L^*\right) \pi + \left(1 - \mu_C^*\right)\left(L^* + \left(1 - \mu_C\right) L\right) \pi - \left(1 - \tau^*\right) \phi\left(\mu_C^*\right) \end{aligned}$$

2.3 Equilibrium innovation and growth

The research arbitrage equations that determine the innovation rates in equilibrium, are simply obtained by taking the first order conditions for each of the above expected profit minus R&D cost expression. Innovation rates in the domestic country thus satisfy:

$$\begin{aligned}(1-\tau)\,\phi'(\mu_A)/\pi &= (\gamma-1)(L+L^*)+\mu_A^*L^* \\ (1-\tau)\,\phi'(\mu_B)/\pi &= (\gamma-1)L+\mu_B^*L+(1-\mu_B^*)\,\gamma L^* \\ 1-\tau\phi'(\mu_C)/\pi &= (1-\mu_C^*)\,L\end{aligned}$$

and symmetrically for innovation in the foreign country.[4]

In steady state, there will be a constant fraction of sectors in each state, q_A, q_B and q_C, with $q_A+q_B+q_C=1$, while aggregate productivity will be

$$\widehat{A}_t = q_A\widehat{A}_{At}+q_B\widehat{A}_{Bt}+q_C\widehat{A}_{Ct}$$

where for example $\widehat{A}_{At}$ is the average productivity level in sectors where the lead resides in the home country. It follows that the growth rate of aggregate productivity (and hence of each country's national income) in steady state will be

$$g=\eta_A g_A+\eta_B g_B+\eta_C g_C \tag{14}$$

where for each state $S=A,B,C$, $\eta_S=q_S\widehat{A}_{St}/\widehat{A}_t$ is the share of aggregate productivity accounted for by sectors in state S in the steady state, and g_S is the expected growth rate of the leading technology $\widehat{A}_{it}$ in each sector currently in state S.

Since the η's add up to one, this implies that the steady-state growth rate of the open economy is a weighted average of the productivity growth rates g_S. These are respectively

$$\begin{aligned}g_A &= (\gamma-1)\,\mu_A \\ g_B &= (\gamma-1)(\mu_B+\mu_B^*-\mu_B\mu_B^*) \\ g_C &= (\gamma-1)\,\mu_C^*\end{aligned}$$

Our conclusions in the remaining part of the section will be derived from comparing the home country research arbitrage equations under openness with the closed economy research arbitrage equation (7), which we reproduce here for convenience:

$$(1-\tau)\,\phi'(\mu)/\pi=(\gamma-1)\,L. \tag{15}$$

2.4 Scale and escape entry

Comparing this closed economy research arbitrage equation with the one governing μ_A:

$$(1-\tau)\,\phi'(\mu_A)/\pi=(\gamma-1)(L+L^*)+\mu_A^*L^* \tag{16}$$

[4] That is:

$$\begin{aligned}(1-\tau^*)\,\phi'(\mu_A^*)/\delta &= (1-\mu_A)\,L^* \\ (1-\tau^*)\,\phi'(\mu_B^*)/\delta &= (\gamma-1)\,L^*+\mu_B L^*+(1-\mu_B)\,\gamma L \\ (1-\tau^*)\,\phi'(\mu_C^*)/\delta &= (\gamma-1)(L+L^*)+\mu_C L\end{aligned}$$

we see that when the home country has the technology lead (case A) it will innovate at a faster rate than when it was a closed economy, because the right-hand side of the leader's research arbitrage equation (16) is larger than the right-hand side of the closed economy counterpart (15). This is because of two effects, scale and escape entry.

The scale effect arises because the successful innovator gets enhanced profits from both markets, not just the domestic market, thus giving her a stronger incentive to innovate. This is why (16) has the sum of size variables $L + L^*$ where (15) has just the domestic size variable L.

The escape entry effect arises because the unsuccessful innovator in the open economy is at risk of losing the foreign market to her foreign rival, a risk that she can avoid by innovating. By contrast the unsuccessful innovator in the closed economy loses nothing to a foreign rival and thus does not have this extra incentive to innovate. Of course this is just the same escape entry effect that we saw above in chapter 12 Formally, this effect accounts for the extra term $\mu_A^* L^*$ that appears on the right hand side of (16) but not of (15).

Comparing the closed economy research arbitrage equation (15) to the one governing the home country's innovation rate in a level sector:

$$(1-\tau)\,\phi'\left(\mu_B\right)/\pi = (\gamma - 1)\,L + \mu_B^* L + (1-\mu_B^*)\,\gamma L^*$$

we see the same two effects at work. The term $\mu_B^* L$ is the escape entry effect; by innovating the home entrepreneur can avoid the risk of losing the local market. The term $(1-\mu_B^*)\,\gamma L^*$ is the scale effect; by innovating the home entrepreneur can capture (with some probability) the foreign market as well as the domestic market.

It follows that both μ_A and μ_B will be larger than the closed economy innovation rate μ. The same will be true for the foreign innovation rates μ_C^* and μ_B^*, which will both be larger than the foreign countries innovation rate when it was closed, μ^*.

2.5 The discouragement effect of foreign entry

We show in Aghion and Howitt (2009, Chapter 12) that a country behind the world technology frontier may be discouraged from innovating by the threat of entry because even if it innovates it might lose out to a superior entrant. This is reflected in the research arbitrage equation governing the home country's innovation rate in case C, the case where it is the technological laggard:

$$(1-\tau)\,\phi'\left(\mu_C\right)/\pi = (1-\mu_C^*)\,L$$

If the foreign country's innovation rate is large enough when it has the lead, then the right-hand side of this research arbitrage equation will be strictly less than that of the closed economy equation (15), so we will have $\mu_C < \mu$. This does not have a direct effect on the growth rate (14) because g_C depends only on the leader's innovation rate μ_C^*. That is, in this state the home country is just catching up, not advancing the global technology frontier. However, as we shall see, a fall in μ_C will nevertheless have an indirect effect on growth by affecting the steady state weights η_S in (14), which are the fractions of productivity accounted for by the sectors in each state.

2.6 How trade can enhance growth in all countries.

Suppose first that one country, say the home country, is less innovative than the other, in the sense that its closed economy innovation rate μ is less than the foreign country's innovation rate when closed μ^*. Suppose also that the discouragement effect of foreign entry is so great, that once it is exposed to foreign trade the home country does no research in sectors where it is a laggard. That is, the above research arbitrage equation governing μ_C cannot be satisfied by any positive μ_C, so instead the equilibrium innovation rate of the home country in state C is $\mu_C = 0$. Suppose however that all the other innovation rates are positive in all 3 states.

In this case what will happen is that the home country will eventually lose the technology lead in all sectors. In the steady state the fraction of sectors in state C will become $q_C = 1$, while $q_A = q_B = 0$. Thus according to the steady-state all profit income will be earned in the foreign country, and the growth rate of national income in all countries will be

$$g = g_C = (\gamma - 1)\,\mu_C^*$$

Since μ_C^* is larger than the foreign innovation rate μ^* before opening up to trade, therefore trade will raise the foreign growth rate. Since both countries grow at the same rate in the open economy, and by assumption the foreign country grew faster than the home country before trade, therefore trade will raise the growth rate in the home country as well.

This illustrates how trade can allow productivity growth even in countries that do not innovate, something that was not possible without foreign trade. For example, as we saw in chapter 7 above, a country with a low level of financial development might end up stagnating relative to the rest of the world. In the present model if the policies favoring innovation τ in the home country are perverse enough then the country will not innovate at all when it is closed, and hence will have a zero growth rate when closed. Yet when it is open to trade it will grow as fast as the most innovative country.

The reason for this effect is what Keller (2004) calls the direct channel of technology transfer. Productivity of workers in the final sector in the non-innovating country can grow despite the lack of domestic innovation because they are able to work with imported intermediate products whose quality continues to grow as a result of foreign innovations. In this sense trade can act as a substitute for innovation in some countries while it acts as a spur to innovation in others.

As another example, suppose that neither country innovates when it is a laggard. Then the fractions q_A and q_B of sectors in which each country has a lead will remain constant over time. If the home country has a smaller innovation rate than the foreign country (i.e., $\mu_A < \mu_C^*$) then average productivity of sectors in state C will grow without bound relative to average productivity of sectors in state A, so in the steady state we will again have $q_C = 1$ and $g = (\gamma - 1)\,\mu_C^*$. Again both countries will have their growth rates raised by trade.

Finally, in the case where both countries are identical ($L = L^*$ and $\tau = \tau^*$), then by symmetry the growth rate will be

$$\begin{aligned} g &= (1-\eta_B)\, g_A + \eta_B g_B \\ &= (\gamma-1)\left((1-\eta_B)\,\mu_A + \eta_B\left(2\mu_B - \mu_B^2\right)\right) \\ &= (\gamma-1)\left((1-\eta_B)\,\mu_A + \eta_B\mu_B\right) + (\gamma-1)\,\mu_B\,(1-\mu_B) \end{aligned}$$

which is larger than the closed economy growth rate $(\gamma - 1)\,\mu$ not only because of the scale and escape entry effects that make μ_A and μ_B both larger than μ, which is why the first term on the right hand side of this last expression is greater than $(\gamma - 1)\,\mu$, but also because of the duplication effect. That is, when sectors are level then there are two possible frontier innovators, so even if one fails to advance the frontier the other might; this accounts for the second term on the right hand side of the last line above.

2.7 How trade can reduce growth in one country

The fact that trade raises growth in both countries when either the countries are symmetrical or one country fails to innovate when behind suggests that trade will usually raise growth in both countries. But there can be exceptions. These exceptions of course must involve countries that are asymmetrical. For example, consider the case of a small country (home) whose policies used to be very unfavorable to innovation but which has recently undertaken a reform to make the country more innovative. Suppose these policies have been so successful that just before opening up to trade, the home country has a faster growth rate than the foreign country:

$$\mu > \mu^*$$

but the reforms have been so recent that the home country is still behind the foreign country in all sectors. Then initially after the opening up to trade all monopolies will reside in the foreign country; that is, all sectors will be in case C above. Now suppose furthermore that the discouragement effect is large enough that the home country does not innovate when behind ($\mu_C = 0$). Then as we have seen all monopolies will remain forever in the foreign country.

This is the case in which, as we saw above, the home country's level of national income might actually fall when trade is opened up, because the increased efficiency of the selection effect might be outweighed by the loss of profits from the home-country monopolists that are forced out of business by foreign competition. What we can now see is that whether or not national income falls at first, the home country's growth rate from then on may be lower than if it had never opened up to trade.

More specifically, if it had not opened up for trade then its growth rate would have remained equal to

$$g = (\gamma - 1)\,\mu$$

whereas under open trade its growth rate will be that of each sector in case C, namely

$$g' = (\gamma - 1)\,\mu_C^*$$

So the home country growth rate will be reduced by trade if and only if $\mu_C^* < \mu$. Now we know from our analysis above that μ_C^* must exceed the innovation rate that the foreign country would have experienced under autarky:

$$\mu_C^* > \mu^*$$

but this does not guarantee that it exceeds the innovation rate that the home country would have experienced under autarky. Indeed if μ_C^* is close enough to μ^* then it will be strictly less than μ and the home country's growth rate will indeed be reduced by trade.

This is where our assumption that the home country is small comes into play. For if it is very small relative to the foreign country then the scale effect of trade on the foreign innovation rate μ_C^* will be small. Since we are assuming that the home country never innovates when behind, therefore there is no escape entry effect on μ_C^*, so if the home country is small enough then μ_C^* will indeed be close enough to μ^* that it falls below μ and the home country's growth rate is diminished by trade.

So we have a presumption that if there are instances where trade is bad for growth, they are probably in small countries that start off far behind the global technology frontier. We also have an example of how economic reform needs to be sequenced properly in order to have its desired effect. That is, generally speaking a country's growth prospects are enhanced by liberalizing trade and by removing barriers to innovation. But if these reforms are undertaken simultaneously then their full benefits might not be realized. Instead it might be better to remove the barriers to innovation first and then to wait until several domestic industries have become world leaders before removing the barriers to international trade.

3 Summarizing

In this Appendix we have analyzed the effects of trade openness on productivity growth. First, trade allows the most advanced producer in each sector to sell to a larger market. And by increasing market size, trade increases the size of ex post rents that accrue to successful innovators, thereby encouraging R&D investments. Second, trade raises competition in product markets, which in turn encourages innovation aimed as escaping competition by more advanced firms in the domestic economy. On the other hand, trade liberalization may discourage innovation by laggard firms. This discouragement effect in turn introduces the possibility that trade may sometimes reduce growth, in particular in small countries that are far below the world technology frontier. Our analysis in fact suggests that it might be better to remove barriers to innovation prior to fully liberalizing trade in such countries.